VOLUME 78 SEPARATE No. 125

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

APRIL, 1952

TORSION OF PLATE GIRDERS

By F. K. Chang, J. M. ASCE, and Bruce G. Johnston, M. ASCE

STRUCTURAL DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

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Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y.

Proceedings—Vol. 78 April, 1952 Separate No. 125

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

TORSION OF PLATE GIRDERS

By F. K. Chang,[1] J. M. ASCE and Bruce G. Johnston,[2] M. ASCE

Synopsis

Analytical and experimental research was undertaken at Lehigh University, Bethlehem, Pa., to determine the stress distribution, stiffness, and strength of bolted, riveted, and welded plate girders under uniform twisting moment. Seventy-two tests were made on 15 different specimens. Eleven of the specimens were finally tested into the plastic range.

Except for initial pilot tests of small specimens, all tests were made on the 2,000,000 in.-lb torsion testing machine[3] especially built for this program of research. Test variables included the number of cover plates, the rivet pitch, variable tension in bolts, and a variety of girder cross-section sizes up to 4 ft in depth, with or without stiffeners.

Formulas are proposed for calculating rivet or bolt pitch and size of welds for the bolted, riveted, and welded plate girders, or, when rivet pitch or weld size are given, the torsional stiffness and strength can be predicted.

Introduction

General.—In the design of an open section such as an I-beam, bending is of primary importance and the problem of torsion, if it exists at all, is usually secondary. If torsional loads are of primary concern, a closed box or tube section is generally used.

A knowledge of the torsional behavior of built-up I-section girders is expecially necessary in problems involving lateral buckling, as corroborated by recent investigators in this field. Karl de Vries, M. ASCE, states "Torsion formulas for plate girders do not seem to be available * * *"[4] George Winter,

Note.—Written comments are invited for publication; the last discussion should be submitted by October 1, 1952.

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[3] "A Torsion Testing Machine of 2,000,000 Inch-Pound Capacity," by F. K. Chang, K. Endre Knudsen, and B. G. Johnston, *Bulletin No. 160*, A.S.T.M., September, 1949, p. 49.

[4] "Strength of Beams as Determined by Lateral Buckling," by Karl de Vries, *Transactions*, ASCE, Vol. 112, 1947, p. 1245.

M. ASCE, in a similar paper comments, "The amount of experimental evidence on the torsional behavior of built-up members cannot yet be regarded as sufficient."[5]

The general mathematical treatment of the torsion problem was first made by Saint-Venant[6] in 1855. He also presented particular solutions for constant twisting moment applied to shafts of various cross sections. His solution for the rectangle is directly applicable to the plate girder, which consists principally of an assemblage of narrow rectangles.

Summary of Torsion Theory.—The development of the general torsion theory[6,7] is not within the scope of this paper. In the case of a straight shaft of uniform, circular cross section, the relationship between torsional moment (M) and angle of twist per unit length (θ) is given by

$$M = J\,G\,\theta \qquad (1)$$

in which G is the modulus of elasticity in shear, and J is the polar moment of inertia. In the case of the circular section, plane cross sections before twist remain plane after twist and the resultant shear stress at any point is proportional to the distance from the central axis of the shaft. Neither of these assumptions will hold for the noncircular sections, but the relationship given by Eq. 1 may then be modified to

$$M = K\,G\,\theta \qquad (2)$$

in which K is defined as a torsion constant[8] determined by the shape and dimensions of the cross section. S. Timoshenko[7,9] uses the notation C as equal to $K\,G$, in which C is termed the "torsional rigidity" of the particular section.

In the case of the narrow rectangle having a width w, more than 3 times the thickness t, the solution by Saint-Venant reduces to

$$K = \frac{w\,t^3}{3} - 0.21\,t^4 \qquad (3)$$

In the case of the circular shaft the resultant shear stress τ is given by

$$\tau = \frac{M\,r}{J} \qquad (4)$$

in which r is the radial distance from the central axis of the shaft.

For a rectangular plate the resultant shear stress in the cross section will be parallel in direction to the wide side and will be proportional in magnitude to the normal distance from the middle plane, except near the narrow edges. Away from the narrow edges, at the surface of the broad sides, the maximum shear stress in the narrow rectangle is closely approximated by

$$\tau_{max}\,\frac{M\,t}{K} \qquad (5)$$

[5] "Strength of Slender Beams," by George Winter, *Transactions*, ASCE, Vol. 109, 1944, p. 1321.

[6] "Resistance des Corps Solides," by M. Navier, 3rd Ed., 1864, as edited by Saint-Venant.

[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934.

[8] "Structural Beams in Torsion," by Inge Lyse and B. G. Johnston, *Transactions*, ASCE, Vol. 101, 1936, p. 857.

[9] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., New York, N. Y., 2d Ed., Vol. 2, 1941.

The maximum shear stress parallel with the narrow surface at the ends of the section is always less in magnitude than the maximum stress in the broad sides, hence Eq. 5 gives the maximum stress in the section.[6] When w is more than four times t, as in the case of most components of structural sections, the error in Eq. 3 is negligible and approaches zero as w gets very large in comparison with t.

There have been many investigations of torsion problems, few of which treat the subject of built-up girders, the primary concern of this paper.

The torsion of rolled structural I-beam and wide flange sections had been the subject of an earlier investigation.[8] Formulas were developed for evaluating the torsional constant (K) for rolled sections, but the application of these formulas to built-up members, such as riveted, bolted, or welded plate girders was open to question. The various components of a built-up member may act together as an equivalent solid section or, if not joined, may act separately. The torsional stiffnesses evaluated for these extreme assumptions will be very different. In some practical cases, the equivalent solid section torsion constant (the measure of torsional stiffness) is as much as 15 times as large as the value of K for separate action. In order to determine the magnitude of K for built-up sections, an extensive experimental investigation was considered necessary. I. E. Madsen,[10] M. ASCE, has made a study showing that the torsional constants of small models of riveted and welded built-up I-beams are somewhat less than those of the equivalent solid section.

Because small size specimens with cold driven rivets were used in the tests conducted by Mr. Madsen, the object of the investigation reported on in this paper was to test larger girders and to make an extensive study of the various factors such as pitch of rivets and bolts, size of weld, tension in bolts, and the presence of stiffeners, that might affect the torsional behavior of built-up members.

Built-up Plates

The rectangular section composed of built-up plates will be discussed first because these units are the basic component of the built-up plate girder.

If a stack of n plates, each of equal thickness t, are bolted or riveted together and transmit a torsional moment, the range of behavior will be between that of a solid piece with the thickness $n\,t$ as one extreme and that of n plates twisted separately as the other. The torsional constant of a plate varies as the cube of its thickness. Therefore, built-up plates with equivalent solid section behavior are much stronger and stiffer than built-up plates with separate action. With n as the number of plates, the ratio of the moment of the solid piece to the moment of the stack of separate plates equals n for the same maximum resultant shear stress. For the same angle of twist, the ratio of moments becomes equal to n^2.

For example, a stack of 4 plates, riveted together, will twist 16 times more and be stressed 4 times more if its plates act separately than if it behaves as an equivalent solid section. In actual cases, the built-up plates behave somewhere between these two extremes, depending principally on the pitch, tightness, and

[10] "Report of Crane Girder Tests," by I. E. Madsen, *Iron and Steel Engineer*, November, 1941.

location of gage lines of bolts or rivets. This intermediate behavior will be termed integral behavior in this paper.

Slip Between Different Components.—The different components of the built-up plate will slip longitudinally with respect to each other during twist if loosely bolted together. If this slip can be prevented, integral behavior, as previously defined, will be obtained. The longitudinal slip between plates (assuming no interaction) may be considered as arising from two causes: (1) The slip caused by the longitudinal warping of each individual cross section (independent of the location of the center of twist); and (2) the longitudinal displacements resulting from rigid body rotations. These factors are dependent on the location of the center of twist and an arbitrary location of zero displacement. The shearing stress distribution in the component rectangular sections is independent of the second type of displacement.

Longitudinal warping of the individual cross section may be closely approximated for the narrow rectangular shape of thickness t and width w. At section x-x in Fig. 1(a), a short distance from the narrow edge, the resultant shear will

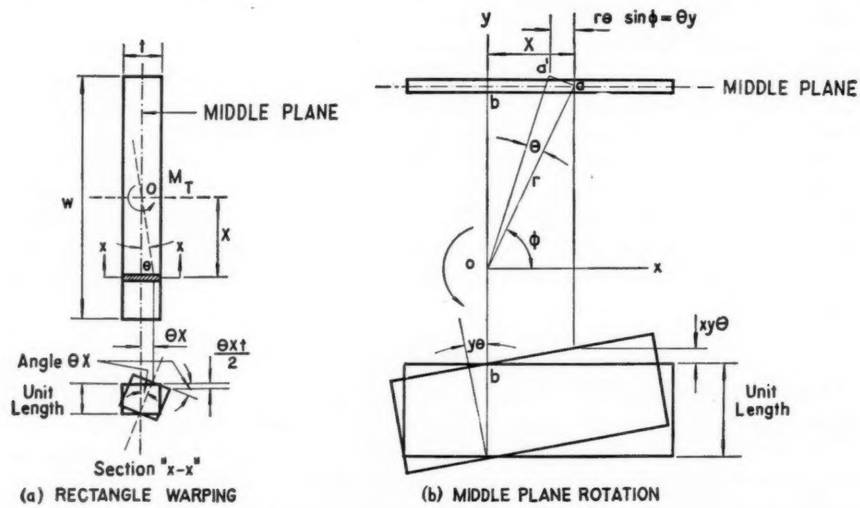


Fig. 1.—Longitudinal Displacement

act nearly parallel to the wide side of the rectangle and the component of shear stress acting parallel to the plane x-x will be of negligible magnitude. Hence, if a unit length of section is twisted through the angle θ, with the center of twist at 0, section x-x must rotate as a rigid body through angle θ_x. For small angles of twist there will be negligible longitudinal displacement of the middle plane of the rectangle, except for regions very near the narrow edges. Hence, there will be longitudinal warping displacements equal in magnitude to $\theta\, x\, t/2$ at the surface of the wide face. Near the narrow edges this simple relationship will no longer be valid and a more rigorous approach is required for precise evaluation.[6] Nevertheless, the warping displacement at the extreme corners will be

closely approximated for the very narrow rectangle by letting $x = w/2$ and, therefore, will equal approximately $\theta\, w\, t/4$.

The longitudinal displacements, when the center of twist is not at the centroidal axis but at any location 0 as shown in Fig. 1(*b*) is next considered. Assume that point b does not move longitudinally and let r be the distance from point 0 to any point a in the middle of the plane of the rectangle. The angle between r and x is denoted ϕ. During the twist θ per unit length, any point a will be displaced transversely a distance $r\,\theta$ to point a′ and the component of this displacement in the middle plane of the rectangle will be $r\,\theta \sin\phi = y\,\theta =$ a constant. Since there is zero shear stress in the middle plane of the rectangle (except near the narrow edges) constant lateral displacement equal to $y\,\theta$ can take place in this plane only by rotation through angle $y\,\theta$, as shown in Fig. 1(*b*). This rotation results in longitudinal displacements of magnitude $x\,y\,\theta$ in the middle plane relative to point b.

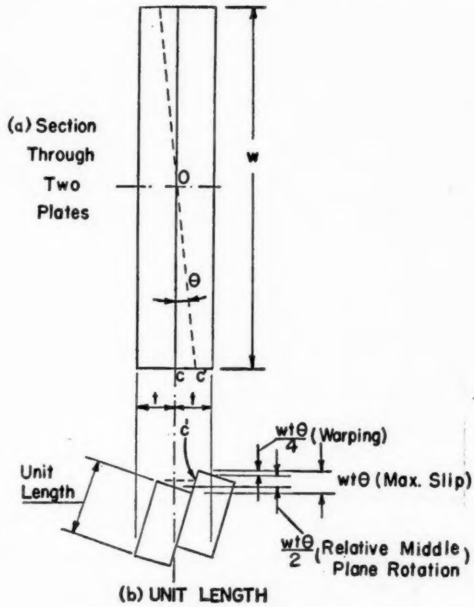


FIG. 2.—SECTIONS IN TWIST

The total longitudinal displacement at any point will be the displacement due to warping added to the displacement due to rotation of the middle plane. In the case of 2 rectangular plates side by side, as shown in Fig. 2, $y = t/2$, hence, at the narrow edge where $x = w/2$, the maximum longitudinal displacement of the middle plane $x\,y\,\theta = \theta\,w\,t/4$ and the relative displacement of the two middle planes is $\theta\,w\,t/2$. If the relative displacement due to warping at point c of Fig. 2 is added to the displacement just obtained for the middle planes, the amount of total slip between the two plates can be determined.

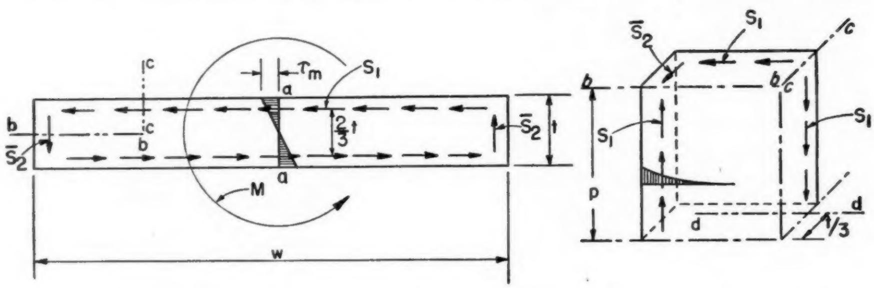
$$\text{Slip} = \frac{\theta\, w\, t}{2} + \frac{\theta\, w\, t}{2} = \theta\, w\, t \qquad (6)$$

This expression applies to any number of plates as long as they are of the same thickness. If the plates are of different thickness, the term t in Eq. 6 should be the average thickness of the two adjacent plates in question.

Pitch of Rivets or Bolts to Develop Integral Action.—If it is possible to eliminate the relative slipping of the several plates when bolted or riveted together, the assemblage of plates would then act integrally. The required shearing forces transmitted by the interfaces between such plates would have to be

furnished in large part by friction developed as a result of the clamping action of the bolts or rivets.

A study of the approximate stress distribution within a solid rectangular section under torsion will permit the determination of the pitch and bolt or rivet size required to transmit the necessary shear forces. Fig. 3(*a*) represents an imaginary section normal to the axis of a rectangular bar of width w and thickness t, transmitting torsional moment M. Away from the narrow sides (as at section a-a) the resultant shear stress is proportional to the distance from the middle plane. Symbol S_1 represents the resultant shear force per unit width of the shear stresses in one direction acting over one half the thickness at section a-a; and S_1 must diminish to zero near the narrow edges but in a thin rectangular section it will be nearly constant over most of the width. Near the narrow edges, the shear stress distribution is complex, but the average resultant



(a) RESULTANT SHEAR FORCE DISTRIBUTION IN NARROW RECTANGULAR PLATE UNDER TORSION

(b) FREE BODY DIAGRAM OF CORNER SEGMENT

FIG. 3.—NARROW RECTANGULAR PLATE UNDER TORSION

force per unit distance parallel to the narrow side is represented by force $\bar{S}_2$ acting near the narrow sides.

From Fig. 3(*a*), it is seen that

$$S_1 = \frac{\tau_m t}{4} \qquad (7)$$

The upper left corner of the section shown in Fig. 3(*a*) is now isolated by longitudinal sections along lines b-b and c-c, as shown in Fig. 3(*b*). From the equivalence of shear stresses acting on planes that are 90° apart, longitudinal shear resultants of magnitude S_1 per unit length must act along section c-c. In the isolated corner section, equilibrium of forces in the longitudinal direction requires that the shear stress resultant along the middle plane section through line b-b must also have an intensity S_1 per unit length.

One half of the resultant torsional couple is supplied by the S_1 forces and the other half is supplied by the forces $\bar{S}_2$ as will be shown. In Fig. 3(*b*), summing the moments about axis d-d

$$\frac{\bar{S}_2 t p}{2} = \frac{S_1 p t}{3} \qquad (8a)$$

or

$$\bar{S}_2 = \tfrac{2}{3} S_1 \qquad (8b)$$

The torque supplied by $\bar{S}_2$ is seen to be a little less than $\bar{S}_2\, w\, t$ and that supplied by the distributed S_1 forces is a little less than $\frac{2\, S_1\, w\, t}{3}$. Hence, introducing the relationship $\bar{S}_2 = \frac{2}{3}S_1$, it is seen that the torques supplied by S_1 and $\bar{S}_2$ are at least approximately equal.

The rectangular section can be assumed to be split into 2 plates along its middle plane (b-b of Fig. 3(a)) and the 2 portions bolted or riveted together at the narrow edges using bolt pitch p. The distributed forces S_1 in the middle plane section through line b-b, over distance p, might then be assumed to be replaced by a concentrated rivet or bolt shear (R)

$$R = S_1\, p \qquad \text{(9)}$$

Substituting S_1 from Eq. 7 into Eq. 9 and denoting the combined thickness of the two plates by T,

$$R = \frac{\tau_m\, T\, p}{4} \qquad \text{(10)}$$

The given thickness T and pitch p determine a definite ratio between the rivet shear R and the maximum torsional shear stress τ_m. If p_r is the rivet pitch that will produce rivet slip R' and shear yield τ_y at the same time, then

$$p_r = \frac{4\, R'}{\tau_y\, T} \qquad \text{(11)}$$

If the pitch is greater than p_r rivet slip will occur before yielding of the material. If the pitch is less than p_r, the material will yield prior to rivet slip. The initial linear relationship between torsional moment and angle of twist per unit length will be limited by slip or yield, whichever occurs first. If the working allowable values of rivet shear stress imply identical factors of safety with respect to slip and yield, respectively, the pitch to produce these values simultaneously will be p_r as given by Eq. 11. Although these factors of safety are usually not equal, the accepted allowable rivet and shear stress values will be used in discussing the design of the specimens tested in this investigation. The rivet pitch for balanced design in pure torsion will be assumed as:

$$p_r = \frac{4\, R_w}{\tau_w\, T} \qquad \text{(12)}$$

in which R_w is the allowable single shear value of the rivet, and τ_w is the permissible shear stress for the material. The effective rivet or bolt clamping distance along the length of the plate must be evaluated to assure that the distributed shear force S_1, acting along the middle plane section through line b-b within the rivet or bolt pitch will be replaced by the rivet or bolt shear. The solution of the problem of a rectangular strip loaded on opposite sides by 2 opposed forces acting in the same line and normal to the surface shows that the compression in the middle plane falls away rapidly and changes to tension at a distance from the line of action of the forces slightly greater than half the plate

thickness.[11] The problem under consideration is somewhat different since it involves three-dimensional effects rather than two-dimensional and since no tension can be developed between the plates. However, it seems reasonable to assume that the effective clamping distance per rivet is equal to the sum of the diameter of the rivet (or bolt) head (A) and the over-all thickness of the assembly (T). Therefore, the rivet or bolt pitch should not be larger than a critical value ($A + T$) or, denoting this critical pitch by p',

$$p' = A + T \quad \text{(13)}$$

The effect of making p larger than p' will be treated in a later section.

When there are more than 2 plates, Eqs. 10, 11, and 12 still apply since the force per unit length (S_1) to be transmitted between the 2 plates nearest the

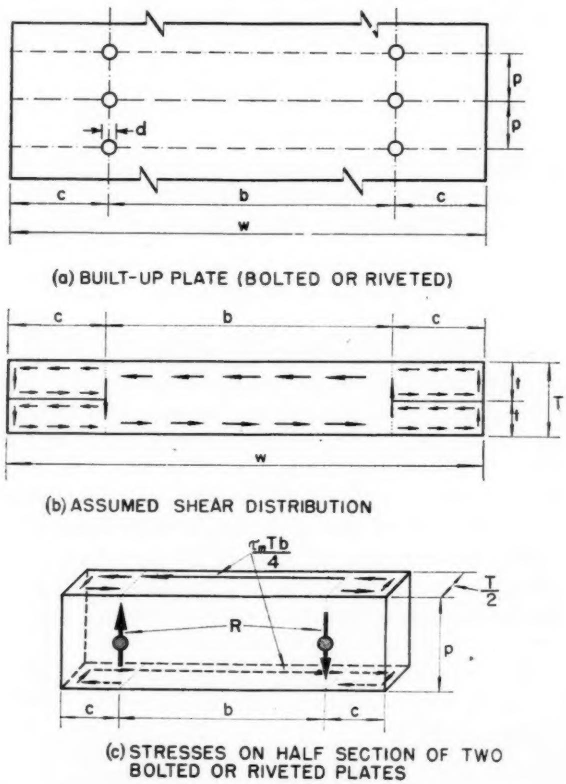


FIG. 4.—BUILT-UP PLATE (BOLTED OR RIVETED)

center will still be given by Eq. 7 (if there are an even number of plates) with the substitution of over-all thickness T in place of single thickness t. If there are an odd number of plates, the force to be transmitted will be somewhat less than S_1 and the procedure will be in error slightly on the conservative side.

[11] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill Book Co., Inc., New York, N. Y., 1934, p. 49.

Evaluation of the Torsion Constant.—When several plates are riveted or bolted together, as shown in Fig. 4, the outer rows of rivets or bolts may be too far from the edges of the plates to produce the pressure required to develop sufficient friction near the edges to replace the longitudinal S_1 forces.

Mr. de Vries[4] assumed that the assemblage between the outer rows of rivets or bolts acts as a solid section, and beyond the line of the assumed solid section, only the contribution of the individual plates would be counted upon. This assumption, as shown in Fig. 4(*b*) seems very reasonable, simple, and on the conservative side. The integral action torsion constant (K_I) can be obtained by modifying Eq. 3 according to the foregoing assumptions. Using the notation in Fig. 5, the torsion constant for n-plates of equal thickness is

$$K_I = \frac{b\,T^3}{3} + \frac{n\,2\,c\,t^3}{3} - 0.21\,T^4 \qquad (14a)$$

or, if the edge effect is neglected,

$$K_I = \frac{b\,T^3}{3} + \frac{n\,2\,c\,t^3}{3} \qquad (14b)$$

If the plates are not of equal thickness, the second term in Eqs. 14*a* and 14*b* should be changed to $\frac{2}{3}c \sum_n t^3$.

Based on the assumed stress distribution in Fig. 4(*b*), the rivet pitch formula may be derived directly from the equilibrium of moments acting on the half section of a combination of 2 bolted or riveted plates having pitch length p, as shown in Fig. 4(*c*). The resultant shear stress on 1 plate, over length b, as previously demonstrated will be $\frac{\tau_m\,T\,b}{4}$. This stress develops a couple about a line normal to the plate interface. This couple is equal to $\frac{\tau_m\,T\,b\,p}{4}$ and is opposed by the couple produced by rivet shear R with moment arm b. Equating the two couples:

$$p = \frac{4\,R}{\tau_m\,T} \qquad (15)$$

This equation is identical with Eq. 11 as previously derived.

When more than 2 plates of equal thickness are bolted or riveted together, as shown in Fig. 5, the single shear force on the rivets at the center plane of the combined unit will be the same as in the case of 2 plates (provided the total number of plates is even) and Eq. 11 for rivet pitch will be unchanged. If there are an odd number of plates of equal thickness, the interface between the central plate and the adjacent plate will not be at the center of the stack and Eq. 11 will be slightly too conservative—the error being about 11% for the case of 3 plates and 4% for

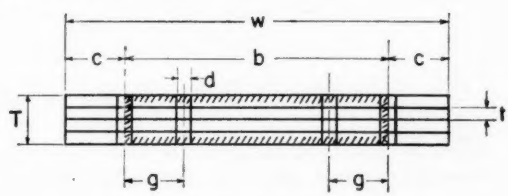


Fig. 5.—Bolted or Riveted Plates

5 plates. In the interest of simplification, for actual design it seems desirable to neglect this discrepancy.

If there are 4 rows of rivets, as shown in Fig. 5, it might be presumed that the two inner rows would not be as effective as the outer rows. Possibly the rivet value of the inner rows should be discounted slightly, or solid section

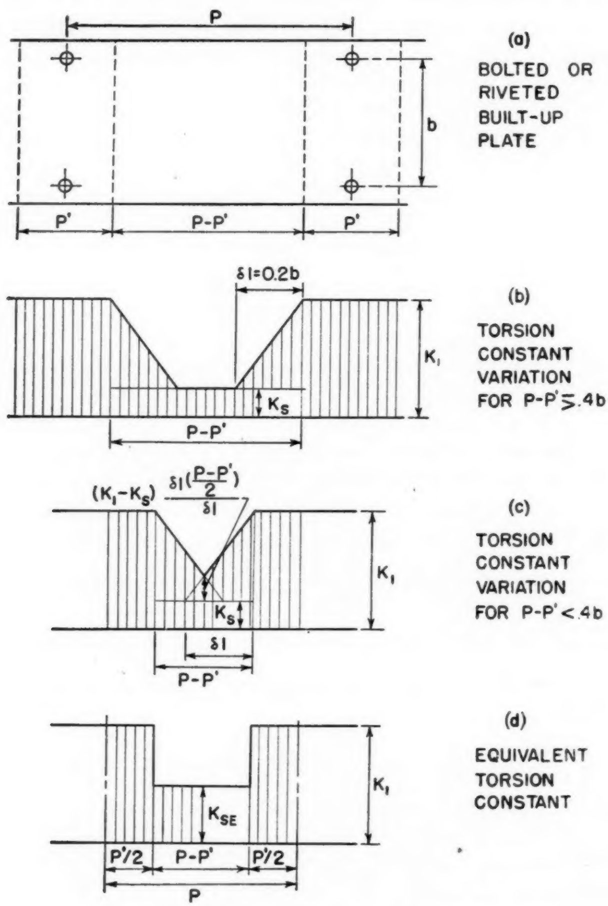


Fig. 6.—Variation of Torsion Constant

behavior assumed only to the mid-distance between the outer pairs of rows. Nevertheless, as will be shown later, tests are in good agreement with the assumption that solid section behavior is maintained between the outermost rows with the innermost rows being assumed equally effective. This procedure should not be extended beyond the limits of the test program outlined in this paper and should be limited, therefore, to a maximum of 4 rows of rivets, with the inner rows as near to the outer ones as the minimum permissible gage (g) will allow.

Loosely bolted together or not bolted or riveted at all, n-plates would act separately and the torsional constant for separate action would be

$$K_s = n\frac{w\,t^3}{3} \qquad (16)$$

If the pitch is larger than the critical pitch (p', Eq. 13), there will be a tendency for the plates to develop separate action in the spaces between the rivet lines. This question also arises in the case of a welded girder having intermittent welds to connect the component parts, in which case no clamping effect should be assumed between the welds and p' is simply the intermittent weld length.

In Fig. 6(a), along the clamped portion (p'), the integral torsion constant (K_I) will be used. Along the portion $(p - p')$ each plate will be assumed to act separately with both ends partially restrained. By ends are meant the dashed lines in Fig. 6(a), at which location integral action is assumed to begin in the riveted or bolted region. For a solid rectangular plate with middle cross section remaining plane during twist, Mr. Timoshenko[12] found that the decrease of ϕ (total angle of twist), resulting from the local constraint is the same as that decrease corresponding to the diminution of the length by the quantity $\delta\, l$—is independent of the length and varies in value from $0.212\,w$ to $0.195\,w$, according to the ratio of w to t. The problem of this paper is somewhat different since, although partially restrained, the ends of the component parts do not remain completely plane, but would have approximately the same degree of warping as the equivalent solid section. However, as approximation, the transition between K_I and K_S will be assumed as shown in Fig. 6, thereby compensating for the partial restraint effect.

In view of the approximations involved, the value of $\delta\, l$ in the present case will be assumed as $0.2\,b$. The equivalent torsional constant along the length $(p - p')$ will be

$$K_{SE} = \delta l\,\frac{K_I + (p - p' - \delta l)\,K_S}{p - p'} = \frac{0.2\,b\,K_I + (p - p' - 0.2\,b)\,K_S}{p - p'} \qquad (17a)$$

for $p - p' \gtrsim 0.4\,b$.

If $p - p'$ is smaller than $0.4\,b$, the torsion constant will be assumed to vary as shown in Fig. 6(c) and the equivalent torsion constant will be

$$K_{SE} = \frac{\dfrac{p - p'}{2}\,K_S + \left[0.4\,b - \dfrac{(p - p')}{2}\right] K_I}{0.4\,b} \qquad (17b)$$

for $p - p' < 0.4\,b$.

The over-all effective torsion constant K_{eff} along the length p can be found by use of Fig. 6(d). By use of Eq. 2,

$$\phi_1 = \frac{M\,p'}{K_I\,G} \qquad (18a)$$

[12] "On the Torsion of a Prism One of the Cross-Sections of which Remains Plane," by S. Timoshenko, *Proceedings*, London Mathematical Society, Vol. 20, 1922, p. 389.

and

$$\phi_2 = \frac{M\,(p - p')}{K_{SE}\,G} \qquad (18b)$$

Then,

$$\phi_p = \phi_1 + \phi_2 = \frac{M\,p'}{K_I\,G} + \frac{M\,(p - p')}{K_{SE}\,G} \qquad (19)$$

in which ϕ_p is the total angle of twist over the length p; ϕ_1 is the total angle of twist over the length p'; ϕ_2 is the total angle of twist over the length $(p - p')$. The average twist per unit length (θ_{aver}) will be

$$\theta_{\text{aver}} = \frac{M}{P\,G}\left[\left(\frac{1}{K_I} - \frac{1}{K_{SE}}\right)p' + \frac{p}{K_{SE}}\right] \qquad (20)$$

since

$$K_{\text{eff}} = \frac{M}{G\,\theta_{\text{aver}}} \qquad (21)$$

Therefore,

$$K_{\text{eff}} = \frac{K_{SE}}{1 - \left(1 - \frac{K_{SE}}{K_I}\right)\frac{p'}{p}} \qquad (22)$$

If p is greater than p', the force distribution shown in Fig. 4(c) is valid only over the distance p', wherein integral behavior is assumed, and the maximum shear stress in this region, for a given rivet stress R is, from Eq. 10,

$$\tau_I = \frac{4\,R}{p'\,T} \qquad (23)$$

If the rivet pitch exceeds p', the calculation of maximum shear stress must also be modified outside of the clamped region in riveted or bolted members, or between intermittent welds in welded members. Within the clamped or welded zone the maximum shear stress may be assumed as

$$\tau_I = \frac{M\,T}{K_I} \qquad (24a)$$

If the term $(p - p')$ is greater than 0.4 b, completely separate action midway between rivets or welds implies that the maximum shear stress at this location is

$$\tau_S = \frac{M\,t}{K_S} \qquad (24b)$$

If the plates are not all of the same thickness, t should be taken as the thickest of the individual plates.

If the term $(p - p')$ is less than 0.4 b the maximum shear stress will be intermediate between τ_I and τ_S. Assuming a linear relationship,

$$\tau_{IS} = M\left[\frac{T}{K_I}\left(1 - \left(\frac{p - p'}{0.4\,b}\right)\right) + \frac{t}{K_S}\left(\frac{p - p'}{0.4\,b}\right)\right] \qquad (24c)$$

Eqs. 24 are advanced tentatively in the belief that they will give conservative strength estimates. Insufficient stress measurements were made regarding the change in stress along the length of the girder to investigate this item in detail.

Effect of Transverse Holes and Stress Concentration.—If a plate containing several open holes is twisted, there will be localized stress concentrations near the holes. In the elastic range these stresses can be calculated approximately by available formulas.[13] If the plate is one of several comprising a built-up bolted or riveted girder, the clamping action of the bolts and rivets, as well as the stress carried through the rivet heads by plate friction, will alter the local stress condition and tend to eliminate the effect of the hole. Furthermore, as in the case of bending of plate girders, initial local yielding in the steel will permit stress redistribution and the holes therefore will have little effect upon the general behavior of the member as a whole.

In the tests to be reported upon, as a limiting case, some of the plate assemblages were tested with zero or nearly zero bolt tension. In such cases, the holes would have their maximum effect upon the torsional stiffness. It was found that a reduced effective width gave good agreement with test results. The reduced effective width is the average width of a solid plate of identical thickness that has the same net volume of material as is in the actual plate with holes deducted. For example, in the case of a plate of width w, having n rows of holes of diameter d and pitch p, the effective width would be

$$w_{\text{eff}} = w - \frac{n \pi d^2}{4 p} \qquad (25)$$

Bolted, Riveted, and Welded Plate Girders

When bolted, riveted, or welded plate girders are twisted, slip between different parts may occur as in the case of built-up plate sections. Therefore, the fabricated girders may not be the equivalent of solid sections, either in stiffness or strength. However, if sufficient rivets, or bolts having adequate tension, or adequate welds are used, it is possible to hold the slippage to a minimum and develop integral behavior intermediate between complete solid section and separate behavior.

Slip Between Different Components.—The maximum slip between different components of fabricated plate girders, if the bolts or rivets are loose, is of comparable magnitude to the slip between any 2 plates of similar size, as given previously by Eq. 6. In a girder of I-section, if there are 2 or more cover plates, the maximum slip between any 2 cover plates would be given directly by Eq. 6. In the case of an angle connecting flange to web of a riveted plate girder, the longitudinal displacement of the middle plane of the angle is determined by the integrated effect of the friction of the component parts adjacent to it. The determination of such slip on the basis of arbitrary hypotheses will not be discussed in this paper as it is of academic interest and the obvious objective in design is to eliminate as much slip as possible by means of bolts, rivets, or welds.

[13] "Strength of Materials," by S. Timoshenko, D. Van Nostrand Co., New York, N. Y., Vol. 2, 2d Ed., 1941, p. 317.

Evaluation of Torsion Constant.—The torsion constant (K) may be determined for rolled sections by standard formulas and procedures.[8] In studying the test results of built-up girders, it is of interest to compute K values for the two extreme conditions of equivalent solid section behavior and separate action behavior, between which the actual integral behavior will lie.

The procedure used in this paper for built-up girders parallels that previously described for the case of built-up plates. Fig. 7 indicates the nomenclature

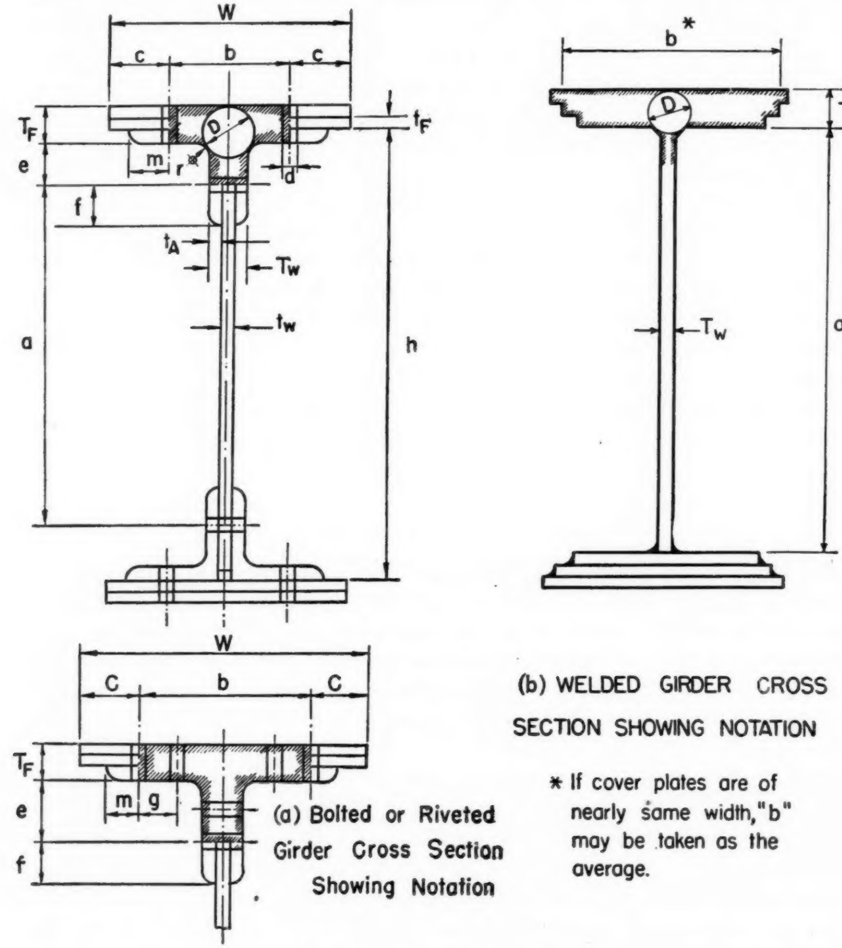


Fig. 7.—Notation for Built-Up Girders

for girder sections. The effective flange width assumed as solid section is the same as that previously discussed in the case of an assemblage of plates, as shown by the shaded areas in Fig. 7.

As noted in the case of the single rectangular plate (Eq. 14a) there is an edge loss (a negative term) in the equation for the torsion constant. As the

ratio of width to thickness increases, this edge loss factor becomes a relatively small item and may be neglected without serious error. When 2 or more rectangles are joined together to form a T-section or I-section, the torsion constant may be obtained by summing the contributions of the separate rectangles, using Eqs. 14*a* or 14*b*, but there is an additional contribution at the juncture or junctures of the various parts that has been called the hump effect.[8] This effect may be evaluated as a function of D^4 (Fig. 7). For the rolled section there is some justification for including both the edge loss and hump effects in evaluating the torsion constant, as was done for the rolled I- and WF-shapes,[8,14] even though these effects tend to offset each other. For riveted or bolted girders however, the uncertainty as to the clamping action of the bolts or rivets makes such refinements unjustifiable. It is also desirable to have as simple a formula as will give an approximation satisfactory for design purposes. This is more important in the case of the plate girder than for the rolled section as in the latter instance sizes are standardized and tables[14] of K-values are available. In view of these facts both the hump and edge effects will be neglected in this analysis. Referring to Fig. 7(*a*) and letting n equal the number of cover plates in each flange, the torsion constant will be

$$K_I = \tfrac{2}{3}\, b\, T^3{}_F + \tfrac{2}{3}\, e\, T^3{}_w + \frac{n\, 4\, e\, t^3{}_F}{3} + \frac{4\,(m+f)\, t^3{}_A}{3} + \tfrac{1}{3}\, a\, t^3{}_w \ldots\ldots (26)$$

If the different parts act separately, the torsion constant, neglecting edge effects will be

$$K_S = \tfrac{1}{3}\left[2\, n\, w\, t^3{}_F + h\, t^3{}_w + 4\left(f + e + m + \frac{(b - t_w)}{2}\, t^3{}_A\right)\right] \ldots\ldots (27)$$

For welded girders (Fig. 7(*b*)) the integral torsion constant can be derived in a similar manner

$$K_I = \tfrac{2}{3}\, b\, T^3{}_F + \tfrac{1}{3}\, a\, t^3{}_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

If the different parts acted separately, with n plates in each flange, the separate action torsion constant (K_S) neglecting the edge effects would be

$$K_S = \frac{2\, n\, b\, t^3{}_F}{3} + \tfrac{1}{3}\, a\, t^3{}_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Rivet or bolt pitch (p_r) may be determined by Eqs. 11 or 12, as developed for the case of built-up plates. Then, the pitch required in the flange to simultaneously develop allowable working rivet shear (R_w) and maximum shear stress due to torsion (τ_m) will be

$$p_r\ (\text{flange}) = \frac{4\, R_w}{\tau_w\, T_F} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30a)$$

if the permissible shear stress is 12 kips per sq in., Eq. 30*a* becomes

$$p_r\ (\text{flange}) = \frac{R_w}{3\, T_F} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30b)$$

with the clamping distance requirement (Eq. 13) that p' (flange) $= A + T_F$.

[14] "Torsional Stresses in Structural Beams," *Booklet S-57*, Bethlehem Steel Company, 1950.

The maximum torsional shear stress in the vertical legs of the angles that connect the web plate to the flange will be less than that in the flanges by the factor T_w/T_F, hence the pitch required in these legs is

$$p_r\,(\text{web}) = \frac{4\,R_w}{\tau\,(\text{web})\,T_w} = \frac{4\,T_F\,R_w}{\tau_m\,T^2_w} \qquad (31a)$$

again, if the permissible shear stress is 12 kips per sq in.,

$$p_r\,(\text{web}) = \frac{T_F\,R_w}{3\,T^2_w} \qquad (31b)$$

with the clamping distance requirement that $p'\,(\text{web}) = A + T_w$.

If the rivet or bolt pitch is larger than the critical value p', the torsion constant should be reduced. Eqs. 17*a*, 17*b*, and 22 for built-up plate case can be applied.

No tests were made in combined bending and torsion. Presumably, in combined bending and torsion, the rivet shear produced by torsion, as given by Eq. 10, could be combined with the rivet shear produced by the over-all shear resultant of the vertical loads. The combined rivet shear should not exceed the permissible rivet value.

For welded plate girders, the sizes of welds should be such as to sustain the maximum shearing stress developed in the welds for solid section behavior. For example, if the size of welds to connect the flange cover plates is desired, a study of the free body diagram of Fig. 3(*b*) will indicate that the strength of the welds required is S_1 pounds per linear in. which equals $\frac{\tau_m\,T_F}{4}$ pounds per linear in. by Eq. 7. If a weld of the size $\frac{S}{8}$ in. is assumed to have an allowable strength of 1.2 S kips per in., corresponding to a shear stress of 13.6 kips per sq in. through the minimum weld section, the size of weld can then be determined by finding

$$S = \frac{\tau_m\,T_F}{4.8} \qquad (32a)$$

For maximum shear stress due to torsion of $\tau_m = 12$ kips per sq in.

$$S = 2.5\,T_F \qquad (32b)$$

The required size of weld is $\frac{S}{8}$ in. If welds connecting the various cover plates are intermittent rather than continuous, Eqs. 17*a* or 17*b* and Eq. 22 may be applied to approximate the effective loss in torsional rigidity, in which case p is the weld spacing and p' is the length of weld.

Strength.—In a rolled section under torsion, the critical shearing stress will occur along the outer surface of the beam and along the fillets where the material is thickest. In built-up structural members, the maximum shear stress may occur somewhere else, especially around the rivet or bolt heads. However, neglecting these local stresses, the approximate shear stress can be determined by Eq. 5, with the proper value of total thickness T substituted for the plate thickness t.

The shearing stress is a function of the thickness of the material and the maximum stress in the flange (τ_F) and in web (τ_w) will be approximately

$$\tau_F = \frac{M\,T_F}{K} \qquad (33a)$$

$$\tau_w = \frac{M\,T_w}{K} \qquad (33b)$$

provided the bolt or rivet pitch is less than p' for longitudinal continuity or in case of a welded girder that welds are continuous.

If the pitch exceeds p', or if intermittent welds are used in a welded girder, the maximum shear stress in the flange will occur midway between the lines of rivets or betweeen the intermittent welds, as the case may be, and may be calculated by Eq. 24. This equation probably gives too conservative an approximation of the strength but the problem will be an unusual one because continuous welds should be used if torsional strength is important.

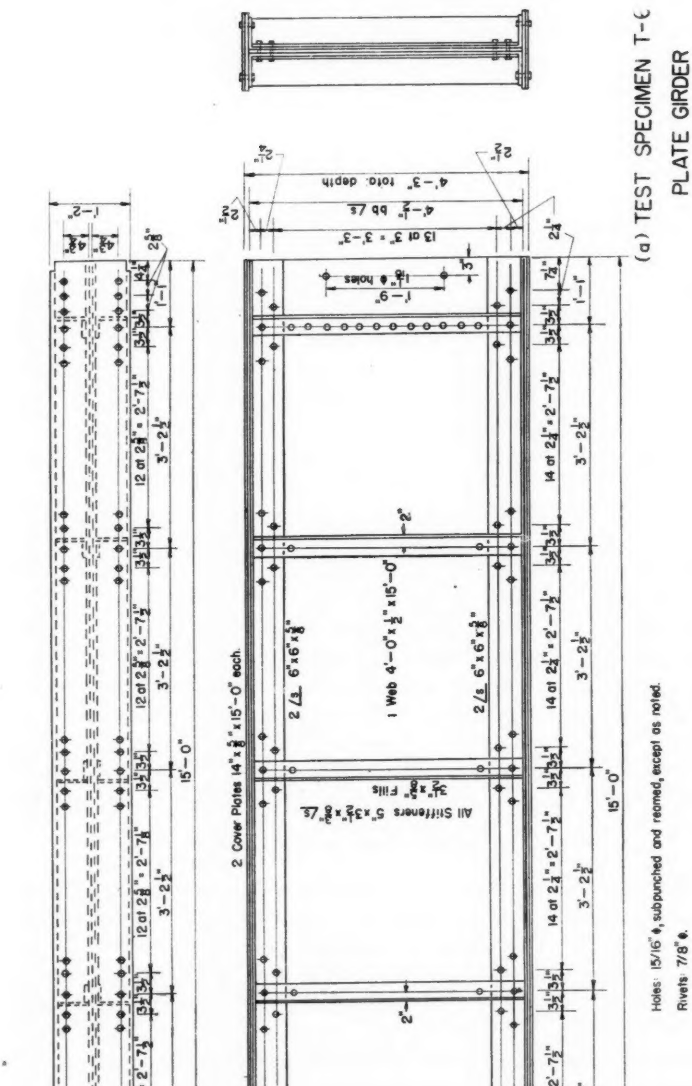
A plate girder of I-beam shape will not be desirable if torsional loads are the primary design factor. The usual need for torsional information will be in connection with lateral instability, when the torsional and lateral bending stiffnesses are of primary concern, and in cases in which the load on the girder primarily causes bending but eccentric application also causes torsion. In lateral instability, the only stress due to torsion, at loads below the critical, will be those unpredictable amounts of stress caused by unavoidable eccentricities in load and the maximum shear stress to be expected is considerably less than 12 kips per sq in. Eqs. 30*b* and 31*b* provide an unduly severe rivet pitch requirement in such a case and should be modified by introducing in Eq. 10 the value of τ_m as required by the actual situation. In combined bending and torsion, as stated before, the rivet pitch would need to be determined so as to properly transmit the combined shear caused by both bending and torsion.

Illustrative Calculation

To illustrate the application of the foregoing equations, the plate girder sections, shown in Fig. 8, of riveted and welded members, respectively, will be used as examples. The numerical results will not agree exactly with those reported in comparison with actual tests as the latter are based on measured dimensions of specimens whereas the following examples will make use of the nominal dimensions.

Illustrative Example: Riveted Plate Girder.—Using the notation of Fig. 7(*a*), the following dimensions are obtained from Fig. 8(*a*):

Symbol	Length in inches	Symbol	Length in inches
a	39.00	t_A	0.625
b	10.00	t_F	0.625
c	2.00	t_W	0.500
d	0.875	T_W	1.750
e	4.125	T_F	1.850
f	1.250		
h	48.50		



(a) TEST SPECIMEN T-6
PLATE GIRDER

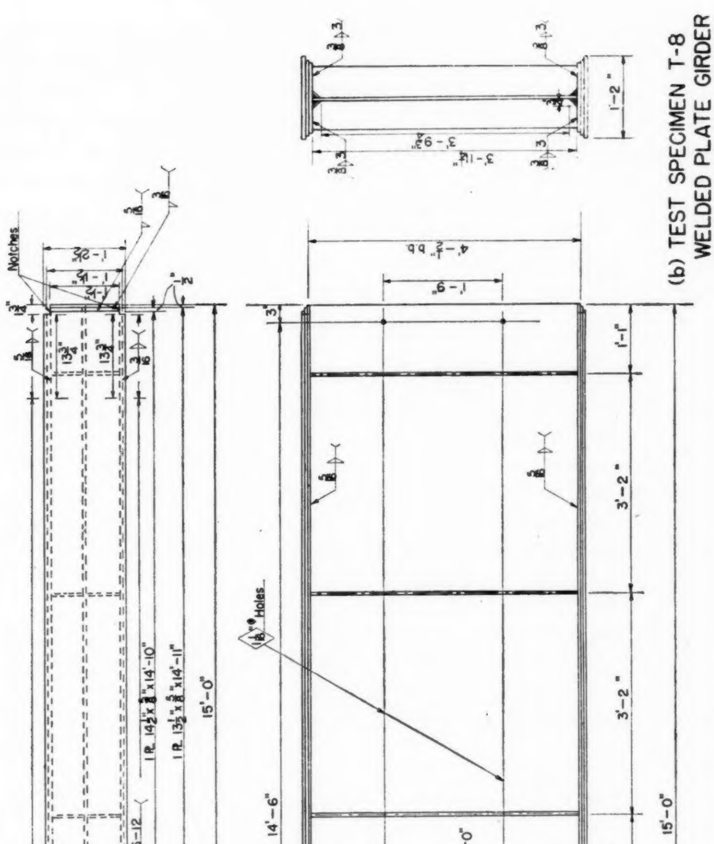


FIG. 8.—PLATE GIRDER SPECIMENS

The rivet pitch in the flange is 2.625 in. The maximum permissible pitch (p') for longitudinal continuity is assumed to be $A + T_F = 1.344 + 1.875 = 3.219$ in., hence no reduction in torsion constant is required. If the working value of a $\frac{7}{8}$-in. diameter rivet is taken as equal to the American Institute of Steel Construction (AISC) permissible of 9.02 kips (single shear at 15 kips per sq in.), Eq. 30*b* gives the flange rivet pitch required to develop 12 kips per sq in. shear stress as p_r (flange) $= \dfrac{9.02}{(3)(1.875)} = 1.60$ in. Since this pitch is less than the actual pitch of 2.625 in. it is obvious that 12 kips per sq in. shear stress in the flange material will not be reached without exceeding the permissible rivet value in single shear. The maximum shear stress in the flange, when the permissible rivet value is reached, will be $\dfrac{(1.60)(12)}{2.625} = 7.31$ kips per sq in.

Whether or not this is too low will depend upon the cause and nature of the torsional loads that are expected and the degree to which these are combined with other loads causing bending. If the problem is simply one of stability during erection there will be no calculable amount of torsional shear and the pitch is probably adequate.

By Eq. 31*b*, the pitch required in the connection between the flange angles and web plate is p_r (web) $= \dfrac{(1.875)\ (0.02)}{(3)\ (1.75)^2} = 1.85$ in.

This also is smaller than the pitch of 2.25 in. that was used, but it is obvious that the pitch used in the flange shows a greater divergence from the optimum requirements and is, therefore, the critical pitch.

The torsion constant for integral behavior is now computed by substitution in Eq. 26:

$$K_I = \tfrac{2}{3} \times 10 \times 1.875^3 + \tfrac{2}{3} \times 4.125 \times 1.750^3 + 2 \times 4 \times 2 \times 0.625^3 + \frac{4\ (1.25 + 1.25)\ (0.625)^3}{3} + \frac{39 \times 0.500^3}{3} = 62.43 \text{ in.}^4$$

This is about 6 times the torsion constant for separate behavior of component parts that can be calculated by Eq. 27.

$$K_S = \tfrac{1}{3}[2 \times 2 \times 14(0.625)^3 + 48.50 \times (0.500)^3 + 4(5.375 + 6)\ (0.625)^3] = 10.28 \text{ in.}^4$$

K_S is calculated for illustrative purposes only and is not needed for design when p is less than p'.

The torsional moment capacity required to develop the calculated maximum shear stress of 7.31 kips per sq in., that occurs in the flange of the girder when the rivets reach their working value in single shear, can be calculated by solving for M in Eq. 33*a*:

$$M = \frac{(7.31)\ (62.43)}{(1.875)} = 243 \text{ kip-in.}$$

The test results confirm the theory that very nearly linear behavior in torsion will be obtained up to torsional moments greater than the calculated value of 243 kip-in.

Illustrative Example: Welded Plate Girder.—Using the dimensions obtained from Fig. 8(*b*), and the notation of Fig. 7(*b*), the following values are obtained:

Dimension	Length in inches
a	45.75
b_{aver}	13.50
t_F	0.625
T_F	1.875
t_W	0.500

The torsion constant for integral behavior is obtained by substitution of the foregoing values in Eq. 28: $K_I = \frac{2}{3} \times 13.50 \times 1.875^3 + \frac{1}{3} \times 47.25 \times 0.500^3 = 61.29$ in.^{4}

Intermittent welds were used in this girder, hence the effective torsion constant must be calculated by Eq. 22 and K_S must also be calculated using Eq. 29:

$$K_S = \tfrac{2}{3} \times 3 \times 13.5 \times 0.625^3 + \tfrac{1}{3} \times 47.25 \times 0.500^3 = 8.56 \text{ in.}^4$$

The distance center to center of welds, or weld pitch (p) is 12 in., and the average length of flange cover plate welds, equivalent to rivet clamping distance (p') is 6.5 in. The term $(p - p') = 5.5$ in. This is greater than $0.4\,b = 5.4$ in., hence, Eq. 17*a* applies and the equivalent torsion constant along distance $(p - p')$ is: $K_{SE} = \dfrac{2.7 \times 61.29 + (5.5 - 2.7) \times 8.56}{5.5} = 34.45$ in.^{4} Then, by Eq. 22, the average effective torsion constant is determined,

$$K_{\text{eff}} = \frac{34.45}{1 - \left(1 - \dfrac{34.45}{61.29}\right) \times \dfrac{5.5}{12}} = 43.10 \text{ in.}^4$$

This example illustrates the value of using continuous welds if torsional rigidity of welded girders is of importance. Continuity would effect an increase in the torsion constant of more than 40%, in this example.

By Eq. 32*b*, the required continuous fillet weld size for balanced torsional strength is $\dfrac{2.5 \times 1.875}{8} = \dfrac{5}{8}$ in. to the nearest eighth inch.

The fillet welds connecting the edges of the two top cover plates are of 3/16-in. size, 6 in. long, and 12 in. center to center. Hence, they do not meet these requirements. The welds will be assumed as having a permissible shear value (in kips per linear inch) of $1.2\ W = (1.2)\ (1.5)$; and W is the weld size in eighths of an inch. Within the welded zone, where integral behavior is assumed, the maximum shear stress in the flange material can be found by solving Eq. 32*a* for the value of τ_m: $\tau_m = \dfrac{4.8\ W}{T_F} = \dfrac{(4.8)\ (1.5)}{1.875} = 3.84$ kips per sq in.

The torsional moment corresponding to the foregoing flange shear stress is found by solving Eq. 24*a* for the value of M; $M = \dfrac{\tau_m K_I}{T_F} = \dfrac{(3.84)\ (61.29)}{1.875}$ $= 125.5$ kip-in.

TABLE 1.—TEST PROGRAM

Designation	Description	Rivet or Bolt: Pitch (in.)	Rivet or Bolt: Gage lines	Bolt torque[a] (ft-lb)
(1)	(2)	(3)	(4)	(5)
	(a) Built-Up Plates P-1, 6-In. by $\frac{1}{4}$-In. Plates (See Fig. 9(a))			
	Single Plate—			
P-1-1	No holes	....	....	....
P-1-2	With holes	$4\frac{1}{2}$	....	....
	Two Plates			
P-1-3	Bolted together	$4\frac{1}{2}$	....	0
P-1-4	Bolted together	$4\frac{1}{2}$	....	30
P-1-5	Bolted together	$4\frac{1}{2}$	....	70
P-1-7	Bolted together	$4\frac{1}{2}$	....	20
P-1-6	Bolted together	$4\frac{1}{2}$	....	130
	Four Plates			
P-1-8	No holes	....	....	....
P-1-9	With holes	$4\frac{1}{2}$	....	....
P-1-10	Bolted together	$4\frac{1}{2}$	....	190
P-1-11	With holes	$2\frac{1}{4}$	....	....
P-1-12	Bolted together	$2\frac{1}{4}$	....	40
P-1-13	Bolted together	$2\frac{1}{4}$	....	100
P-1-14	Bolted together	$2\frac{1}{4}$	....	150
P-1-15	Bolted together	$2\frac{1}{4}$	....	190
P-1-16	Riveted together	$2\frac{1}{4}$	....	....(P)
	(b) Built-Up Plates P-2, 20-In. by $\frac{5}{8}$-In. Plates (See Fig. 9(b))			
	Single Plate—			
P-2-1	No holes	....	....	....(P)
P-2-2	With holes	$2\frac{1}{4}$ (staggered)	....	....
	Two Plates			
P-2-3	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	0
P-2-4	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	150
P-2-5	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	300
P-2-6	Bolted together	$2\frac{1}{4}$ (staggered)	Four inner lines	300
P-2-7	Bolted together	$6\frac{3}{4}$ (center)[b] $2\frac{1}{4}$ (ends)[b]	Four inner lines	300
P-2-8	Bolted together	$6\frac{3}{4}$ (staggered)	Four inner lines	300
	Four Plates—			
P-2-9	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	0
P-2-10	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	15
P-2-11	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	150
P-2-12	Bolted together	$2\frac{1}{4}$ (staggered)	Four outer lines	300
P-2-13	Bolted together	$2\frac{1}{4}$ (staggered)	Four inner lines	150
P-2-14	Bolted together	$2\frac{1}{4}$ (staggered)	Four inner lines	300
P-2-15	Bolted together	$6\frac{3}{4}$ (center)[b] $2\frac{1}{4}$ (ends)[b]	Four inner lines	300
P-2-16	Bolted together	$6\frac{3}{4}$ (staggered)	Four inner lines	300
P-2-17	Bolted together	$2\frac{1}{4}$ (staggered)	Four inner lines	5
P-2-18	Bolted together	$2\frac{1}{4}$ (staggered)	Four inner lines	0
P-2-19	Riveted together	$2\frac{1}{4}$ (staggered)	Four inner lines	....(P)
	(c) 20-In. Built-Up Bolted Plate Girder, T-1A; No Cover Plate (See Fig. 9(c))			
T-1A-1	Bolted	$2\frac{3}{4}$	....	0
T-1A-2	Bolted	$2\frac{3}{4}$	....	100
T-1A-3	Bolted	$2\frac{3}{4}$	....	200
T-1A-4	Bolted	$2\frac{3}{4}$	....	300
T-1A-5	Bolted	$5\frac{1}{2}$	....	300
T-1A-6	Bolted	$8\frac{1}{4}$	....	300
T-1A-7	Bolted	$8\frac{1}{4}$ (center) $5\frac{1}{2}$ (ends)	....	300
T-1A-8	Bolted	$5\frac{1}{2}$ (center) $2\frac{3}{4}$ (ends)	....	300

[a] Specimens all tested in the elastic range except those designated "(P)" which were also tested in the

TABLE 1.—(CONTINUED)

Designation (1)	Description (2)	Rivet or Bolt: Pitch (in.) (3)	Rivet or Bolt: Gage lines (4)	Bolt torque[a] (ft-lb) (5)
(*d*) 20-In. Built-Up Bolted Plate Girder, T-2A; One Cover Plate (See Fig. 9(*c*))				
T-2A-1	Bolted	2¾	....	15
T-2A-2	Bolted	2¾	....	150
T-2A-3	Bolted	2¾	....	300
T-2A-4	Bolted	5½ (center) 2¾ (ends)	....	300
T-2A-5	Bolted	8¼ (flange) 5½ (web)	....	300
7-2A-6	Bolted	8¼	....	300
(*e*) 20-In. Built-Up Bolted Plate Girder, T-3A; Two Cover Plates (See Fig. 9(*c*))				
T-3A-1	Bolted	2¾	....	100
T-3A-2	Bolted	2¾	....	300(P)
(*f*) 20-In. Built-Up Bolted Plate Girder, T-5A; Three Cover Plates (See Fig. 9(*c*))				
T-5A-1	Bolted	2¾	....	30
T-5A-2	Bolted	2¾	....	150
T-5A-3	Bolted	2¾	....	300
T-5A-4	Bolted	2¾ (web) 8½ (flange[c])	....	300
T-5A-5	Bolted	2¾ (web) 8½ (flange)	....	300
(*g*) 50-In. Built-Up Bolted Plate Girder, T-6A (See Fig. 8(*a*))				
T-6A-1	Bolted, no stiffeners	See Fig. 8(*a*)	....	100
T-6A-2	Bolted, no stiffeners	See Fig. 8(*a*)	....	300
T-6A-3	Bolted, with stiffeners	See Fig. 8(*a*)	....	100
T-6A-4	Bolted, with stiffeners	See Fig. 8(*a*)	....	300
(*h*) 50-In. Built-Up Riveted Plate Girder, T-B-1 (See Fig. 9(*c*))				
T-1B-1	Riveted	See Fig. 9(*c*)	....	....(P)
T-2B-1	Riveted	See Fig. 9(*c*)	....	....(P)
T-4B-1	Riveted	See Fig. 9(*c*)	....	....(P)
T-5B-1	Riveted	See Fig. 9(*c*)	....	....(P)
(*i*) 50-In. Built-Up Riveted Plate Girder, T-6B-1 (See Fig. 8(*a*))				
T-1B-1	Riveted, with stiffeners	See Fig. 8(*a*)	....	....(P)
(*j*) 20-In. Welded Plate Girder, T-7 (See Fig. 9(*d*))				
T-7-1	Welded, no cover plate	....	....	....
T-7-2	Welded, one cover plate	....	....	....(P)
(*k*) 50-In. Welded Plate Girder, T-8 (See Fig. 8(*b*))				
T-8-A1	No cover plate and no stiffener	....	....	....
T-8-A2	Stiffeners but no cover plate	....	....	....
T-8-B2	Two cover plates, with stiffeners	....	....	....(P)
Rolled Section T-9; 18WF77 (See Bethlehem Manual of Steel Construction)				
T-9	Rolled Section	....	....	....

Plastic Range. [b] Staggered. [c] At the mid-third section of the flange.

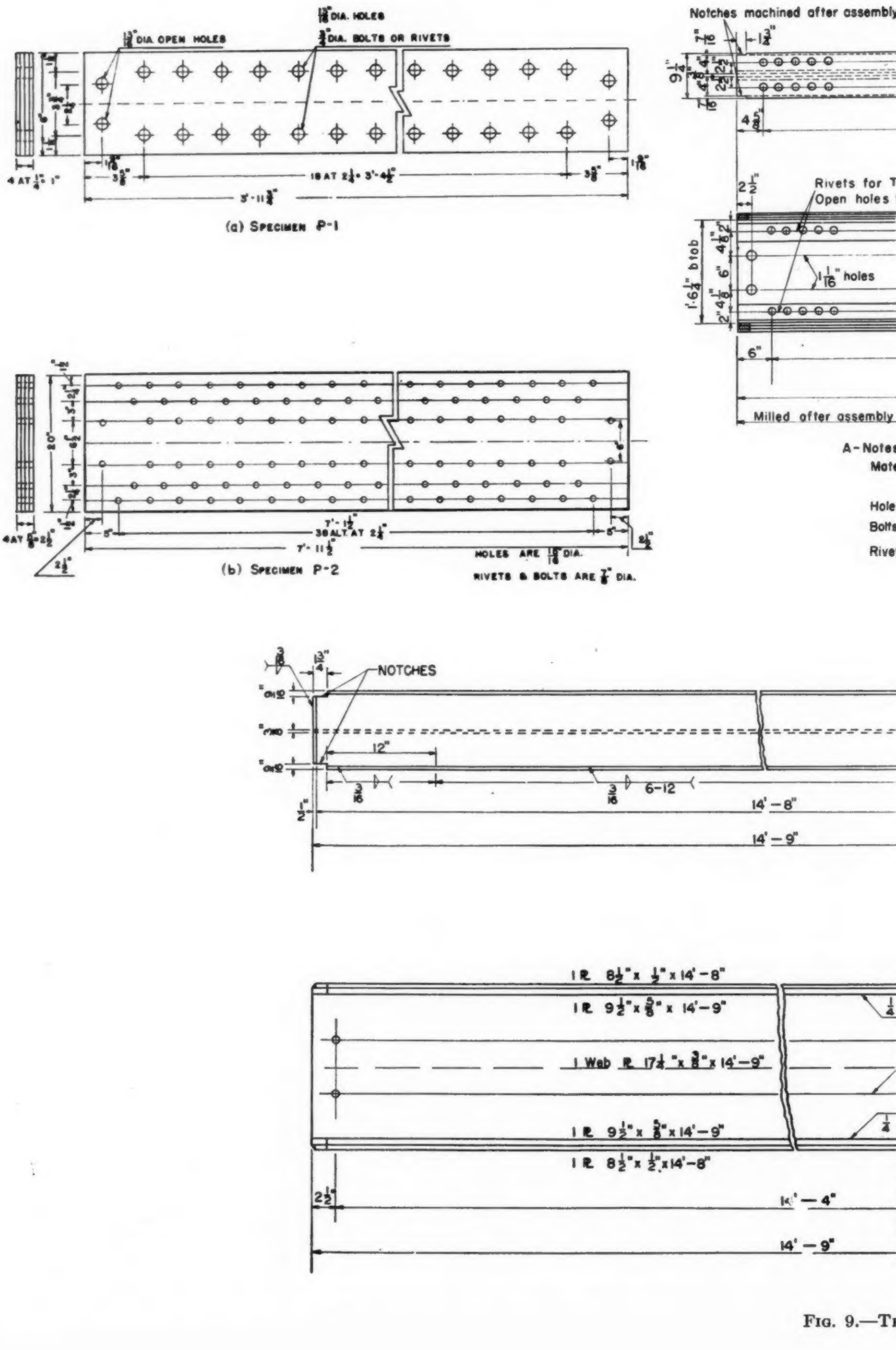
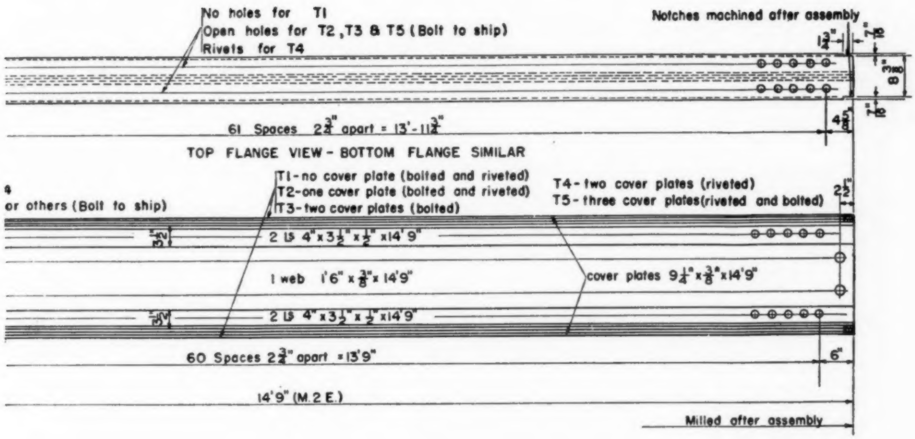

DIA. OPEN HOLES
DIA. HOLES
DIA. BOLTS OR RIVETS
3'-11¾"
(a) SPECIMEN P-1
HOLES ARE $\frac{15}{16}$" DIA.
RIVETS & BOLTS ARE $\frac{7}{8}$" DIA.
7'-11½"
(b) SPECIMEN P-2
Notches machined after assembly
Rivets for T
Open holes f
$1\frac{1}{16}$" holes
Milled after assembly
A-Notes
Mate
Holes
Bolts
Rivet
NOTCHES
12"
6-12
14'-8"
14'-9"
1 PL $8\frac{1}{2}$" x $\frac{1}{2}$" x 14'-8"
1 PL $9\frac{1}{2}$" x $\frac{5}{8}$" x 14'-9"
1 Web PL $17\frac{1}{4}$" x $\frac{3}{8}$" x 14'-9"
14'-4"

FIG. 9.—TEST



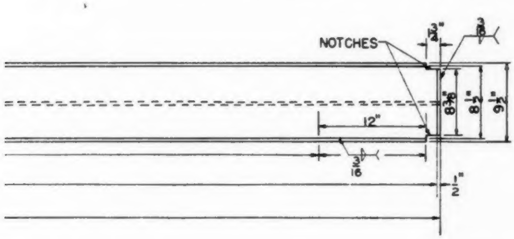
rial: Structural carbon steel ASTM-A7. All plates of one thickness to be made from the same rolling. The materials for the angles and plates must have nearly the same physical properties.

: 15/16" ϕ, subpunched and reamed, except as noted.

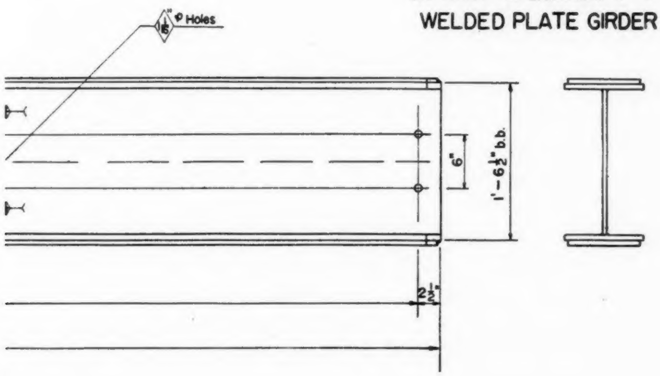
7/8" ϕ, high strength, yield point 70,000psi. minimum, with hexagonal nuts.

s. 7/8" ϕ, report the temperature of the rivets when driven and the manner of driving.

(c) TEST SPECIMENS – T1, T2, T3, T4, and T5. (one each) – BUILT UP I BEAMS



(d) TEST SPECIMEN T-7
WELDED PLATE GIRDER



SPECIMENS

Since p' is greater than 0.4 b, the maximum shear stress in the flange, midway between the welded zones, is given by Eqs. 24. Limited stress measurements indicate that the actual stress between the welds is lower than that given by Eq. 24b. However, the equation is conservative and may be used for design purposes:

$$\tau_S = \frac{(125.5)(0.625)}{8.50} = 9.21 \text{ kips per sq in.}$$

Although this value is 2.4 times the maximum shear stress in the welded zone, it is less than the allowable value of 12 kips per sq in. that has been assumed in these examples. Hence, the most critically stressed parts of the girder are the welds, on the basis of which the torsional moment of 125.5 kip-in. was evaluated. The test girder on which this illustrative example is based showed linear behavior for values of torsional moments up to more than twice this amount.

Torsional Behavior Above the Elastic Range

If the torsional moment exceeds the value causing initial yielding of the material, there is a gradual increase of the yielded zone and the amount of twist per unit increment of torsional moment progressively increases. The elastic-plastic behavior of structural members in torsion has been treated by others, [15,16,17] usually on the basis of no strain-hardening. Mr. Chang has discussed the plastic behavior of the specimens tested in this investigation. Because of the great twisting deformations of girder sections at torsional moments causing initial inelastic behavior, a presentation of this theory is not of practical importance to the design engineer.

When the angle of twist becomes large, longitudinal fibers away from the axis of twist become helices and tend to shorten in proportion to the distance from the twist axis. Hence compressive forces are induced at the center of the section and tensile forces at the sections farthest removed from the twist axis. The tensile forces, acting at the largest angle to the twist axis, have a torsional component about the axis of twist resisting the applied torsional moment. The effect is negligible for small twist angles but becomes considerable in the early stages of plastic yielding. As a result, torsional moments for complete plastic yielding, as computed by the sandheap analogy,[15] give values of torsional moment strength that are too low for the usual structural section. M. S. G. Cullimore [18] has derived formulas for the direct stresses induced during a large twist of I-beam sections. Mr. Chang[19] developed similar formulas and measured these stresses experimentally.

[15] "Plasticity," by A. Nadai, McGraw-Hill Book Co., Inc., New York, N. Y., 1931.

[16] "The Torsion of Solid and Hollow Prisms in the Elastic and Plastic Range by Relaxation Methods," by F. S. Shaw, *Report 11*, Australian Council for Aeronautics, November, 1944.

[17] "On Torsion of Plastic Bars," by F. G. Hodge, Jr., *Journal of Applied Mechanics*, Vol. 16, No. 4, December, 1949, p. 399.

[18] "The Shortening Effect—A Non-linear Feature of Pure Torsion," by M. S. G. Cullimore, *Engineering Structures-Research Engineering Structures Supplement*, Colston Papers, Academic Press, New York, N. Y., 1949.

[19] "Torsion of Built-Up Structural Members," by F. K. Chang, thesis presented to Lehigh University, Bethlehem, Pa., in 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Test Program, Specimens, and Test Apparatus

As the scale factor in bolted, riveted, and welded girders may be important, experimental work on full-size specimens was necessary to verify formulas. Table 1 gives a list of the specimens tested in the complete test program. For bolted types the variables were bolt tension, pitch, and gage lines. The details of all the specimens are shown in Figs. 8 and 9. Space limitations permit presentation of only a part of the test results.

Fig. 10.—Torsion Testing Machine

Design of Specimens.—In general, the specimens tested were not specifically designed for torsion, but rather were intended to have proportions that are common for plate girders designed for normal vertical loads that cause bending in the plane of the maximum moment of inertia. Nearly minimum bolt or rivet pitch was used in many cases, but tests were also made with alternate lines of bolts removed. A detailed analysis of the predicted torsional properties of all the various specimen combinations that were tested and are as listed in Table 1 is not within the scope of this paper. However, the results of many such analyses are shown by the plotted theoretical torque-twist curves in the graphical presentation of test results. The procedure of analysis has been outlined in detail in the two illustrative examples that were based on test specimens T6B and T8B.

The AISC allowable single shear value for rivets was used in the analysis of both the riveted and bolted specimens. The test results indicated that the high tensile strength bolts used, when tightened to 300 ft-lb torque, very nearly simulated the behavior of rivets in the elastic range and in the early stages of yielding.

Specimens P1 and P2 were used to study the behavior of built-up plates. These plates simulate the flange of a heavy girder, from which most of the girder's stiffness and strength in torsion is derived.

With one exception (Test T3A2), the bolted girders were tested only in the elastic range, in order to study the effect of various combinations of plates,

rivet pitch, and line and row spacing to the best advantage. Then, after the bolted tests, the specimens were riveted in the shop and returned to the lab for final testing to failure.

Although the rivet pitch in specimens P1, P2, and T1 to T6 is in many cases less than that required for full torsional strength, the pitch is usually adequate or nearly adequate to provide longitudinal continuity. In such a case full torsional rigidity should be obtained at low torsional loads but general inelastic behavior due to slip, yield, or both, will start at lower levels than for a balanced design.

The rivets were driven using a 50-ton bull riveter with a pressure of 100-lb gage. The rivets were at a temperature of 1650° F (cherry red) when driven and the approximate driving time was 0.05 min per rivet.

The 2 welded girders (specimens T7 and T8) had welds ample for shear stresses that would result from vertical loads causing bending. As shown in the illustrative example, these specimens were underdesigned in strength and longitudinal continuity when tested for pure torsion.

Test Apparatus.—The torsion testing machine, as shown in Fig. 10, was designed and constructed especially for this project and has been described elsewhere.[3] The torque is applied to the specimen by an end plate attached to an 88-in diameter rotating head that is turned by means of a 4-in. by $\frac{1}{4}$-in. flat wire rope, using an old standard model testing machine as the power source. To measure the applied torque accurately, calibrated aluminum torque tubes of various capacity, mounted with SR-4 gages, are inserted at one end of the stationary head. The stationary head, in turn, while resisting the applied torque, rests on rollers that permit longitudinal movement when the specimen shortens during the twist.

Measurements of deformation were made by the means listed in the following tabulation:

Measurement	Apparatus
Strain	SR-4 gages
Angle of twist	Level bars
Variation in length during twist	Ames dials
Slip between different components	Whittemore gages

Adjustable-level bar seats were placed at several stations along the length of each girder. The difference of tilt angle between 2 level bar stations, divided by the distance between them, gives the angle of twist per unit length.

In all tests reported herein, approximately free-end conditions were obtained. Torque is applied by end fixture plates as shown in Fig. 10. The ends are free to warp, except for friction forces that are small in proportion to the forces required to fully restrain the section against warping. Local end-effects caused by the manner of application of the torque taper off very rapidly. The center portion of the girder, therefore, was assumed to be in a state of uniform torsion.

Test Results

Physical Tests of Materials in Test Specimens.—The tensile properties of every part of each specimen were determined. In general, the material met the

American Society for Testing Materials (ASTM) specification requirements for A-7 steel for bridges or buildings. In computing theoretical twists per unit length per unit torsional moment, a representative value of shear modulus (G) was taken as 11,450 kips per sq in., to correspond with a value of $E = 29{,}500$ kips per sq in. and Poisson's ratio value of 0.29.

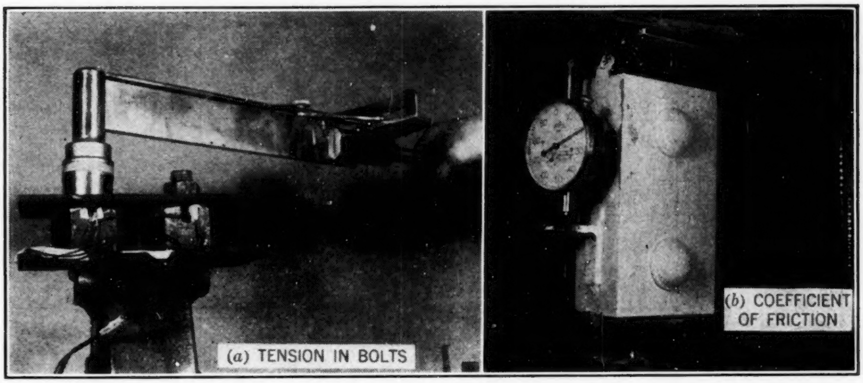


FIG. 11.—BOLT AND RIVET TESTS

Bolt and Rivet Tests.—The tension in the bolts was determined as a function of applied bolt torque by mounting SR-4 strain gages on the shank of bolts in the manner shown in Fig. 11(*a*). By this means, the strain in the bolt can be measured and the tensile force developed in the bolt can be computed for various applied torques. A curve typical of these test results is shown in Fig. 12. Then, the effective coefficient of friction between the plates or between the bolt head, nut, or washer and plate was determined by using a set-up similar to that shown in Fig. 11(*b*) for a riveted specimen. This coefficient depends on the amount, composition, and the condition of dryness of the paint, the roughness of the surface, and other such factors. For ordinary unpainted structural steel, an average value of 0.25 was obtained for the coefficient of friction. The test results are summarized in Table 2.

TABLE 2.—EFFECTIVE COEFFICIENT OF FRICTION FOR BOLTED SPECIMENS

Torque applied (ft-lb)	Unit stress (lb per sq in.)	Total stress (lb)	Average load, to start slip[a] (lb)	Effective coefficients of friction
100	17,000	10,000	5,100	0.25
200	35,000	21,000	10,900	0.26
300	50,000	30,000	16,900	0.28

[a] Double shear bolt value for friction.

Tests were made to determine the values of coefficients of friction for rivets in single shear for conditions typical of those used in manufacturing the test specimens. The test results for the various conditions of equipment, air pressure, temperature, and driving time are summarized in Table 3. These

specimens allowed 0.5-in. slip before coming to bearing, thus differentiating clearly between the effects of friction and bearing resistance.

Plate Specimens.—These were a preliminary to girder tests and only a representative selection of test results are presented in this paper.

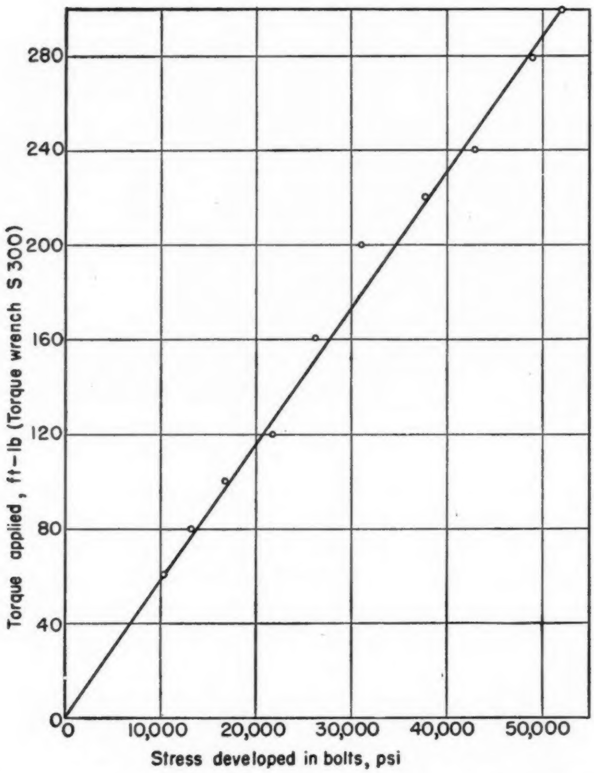


FIG. 12.—TYPICAL TORQUE-TENSION CURVE FOR $\frac{3}{4}$-INCH HIGH STRENGTH BOLTS

The 6-in. × $\frac{1}{4}$-in. plates (specimen P1, Fig. 9(*b*)) were pilot tests in a standard 24,000 in.-lb torsion testing machine. All other specimens were tested by using the 2,000,000 in.-lb machine. In the tests there were 2 rows of bolts, $\frac{3}{4}$ in. diameter at $2\frac{1}{4}$ in. center to center. The effect of bolt tension on both the strength and stiffness of a stack of 4 plates is shown in Fig. 13, with the curve for riveting included for comparison. The test result for 2 plates bolted together is almost identical to that of the 4-plate case.

Four 20-in. × $\frac{5}{8}$-in. plates (specimen P2, Fig. 9(*b*)) bolted together, were twisted in the elastic range only. The effects of bolt torque and gage line location are shown in Fig. 14(*a*). The longitudinal slip between plates during twist was also measured and was found to depend largely on the bolt torque. The longitudinal slippage between loosely bolted plates checked very well with theoretical curves as determined by Eq. 6.

Four plates riveted together were tested to complete failure. The torque-twist and torque-slip curves of the test are shown in Fig. 14(*b*). Strain lines appeared on the rivet heads at 240,000 in.-lb torque, on the plate near the rivet heads at 301,000 in.-lb, and in the portion between the two outer rows of rivets at 326,500 in.-lb.

TABLE 3.—RESULTS OF TESTS ON RIVETS FOR SPECIMEN T6

Test number	Driving equipment	Air pressure (lb per sq in.)	Approximate rivet temperature (degrees Fahrenheit)	Approximate driving time (min)	Load at start of slip[a] (lb)
S1a	Bull riveter[b]	100	1650[c]	0.05	32,000(T6)
S1b	Bull riveter	100	1650[c]	0.05	20,000(T1 to T5)
S2	Hand gun	100	1650[c]	0.05	19,000
S3	Bull riveter[b]	80	1650[c]	0.05	20,000
S4	Bull riveter[b]	100	1900[d]	0.05	30,500
S5	Bull riveter[b]	100	1650[c]	0.25	35,000

[a] Double shear value. [b] 50 ton model. [c] Cherry red. [d] Bright cherry red.

Rolled Section (*Specimen T9*).—One rolled section was tested in order to compare its characteristics with the bolted, riveted, and welded types and to correlate this program with earlier investigation.[8] The test results were shown to be in good agreement with this former research. The torque-twist curve is shown in Fig. 15(*a*). Strain lines first appeared along the fillets at 75,000 in.-lb

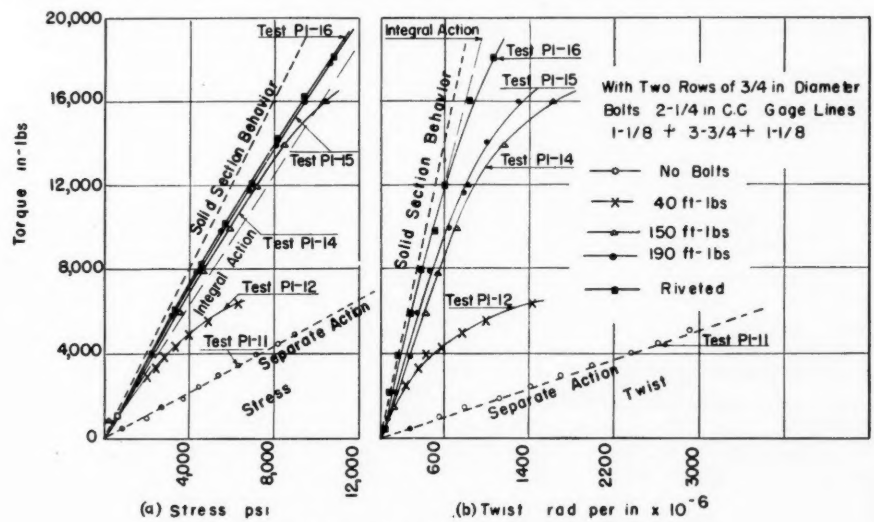
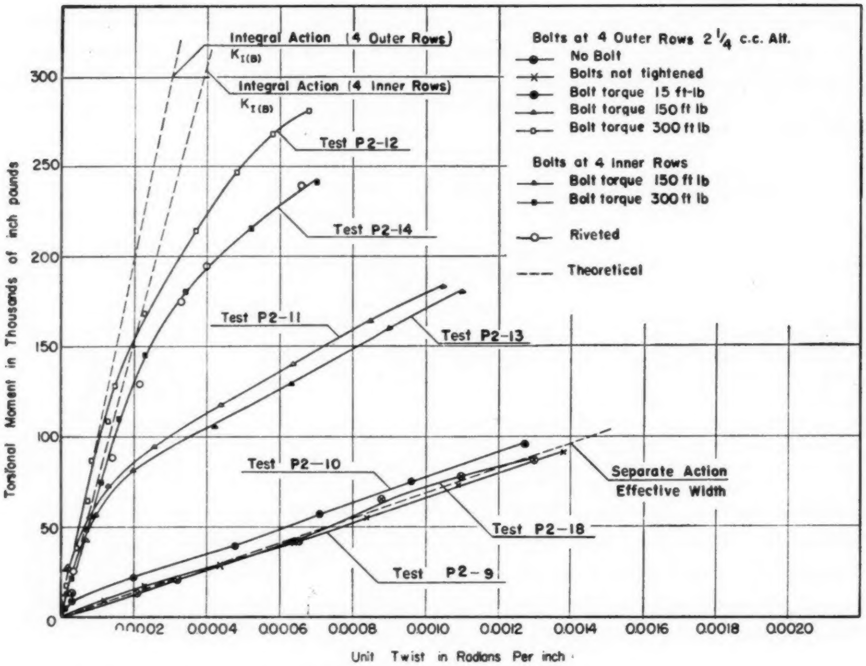
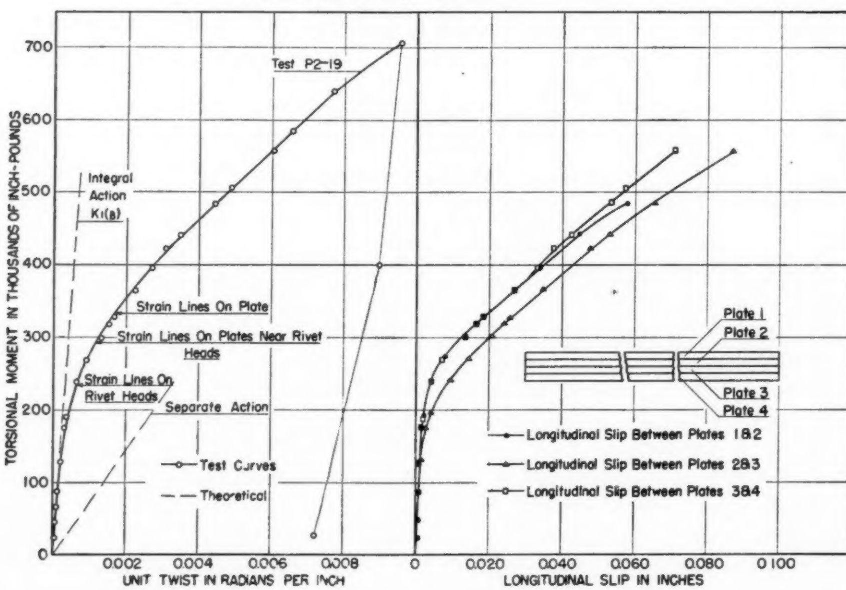


FIG. 13.—TEST RESULTS FOR 4 PLATES BOLTED TOGETHER

where stress concentrations occurred due to the sharp curvature of the fillet. At 86,000 in.-lb strain lines appeared along the center line of the flange where the largest inscribed circle touches the boundary. The moment for the completely plastic state, assuming no strain-hardening, is also shown, but it is noted



(a) EFFECT OF BOLT TENSION & GAGE LINE LOCATIONS ON FOUR 20×5/8 PLATES



(b) TORQUE TWIST AND TORQUE SLIP CURVES ON FOUR 20"×5/8" PLATES RIVETED TOGETHER UNDER TORSION (SPECIMEN P-2)

FIG. 14.—CHARACTERISTIC CURVES FOR ASSEMBLY OF 4 PLATES

that even at low unit twist angle the beam offers a much higher torsional resistance caused by the development of longitudinal normal stresses.[19]

Bolted Plate Girders.—All bolted specimens except T3 were tested in the elastic range only. Different bolt pitches and bolt torques were used in these

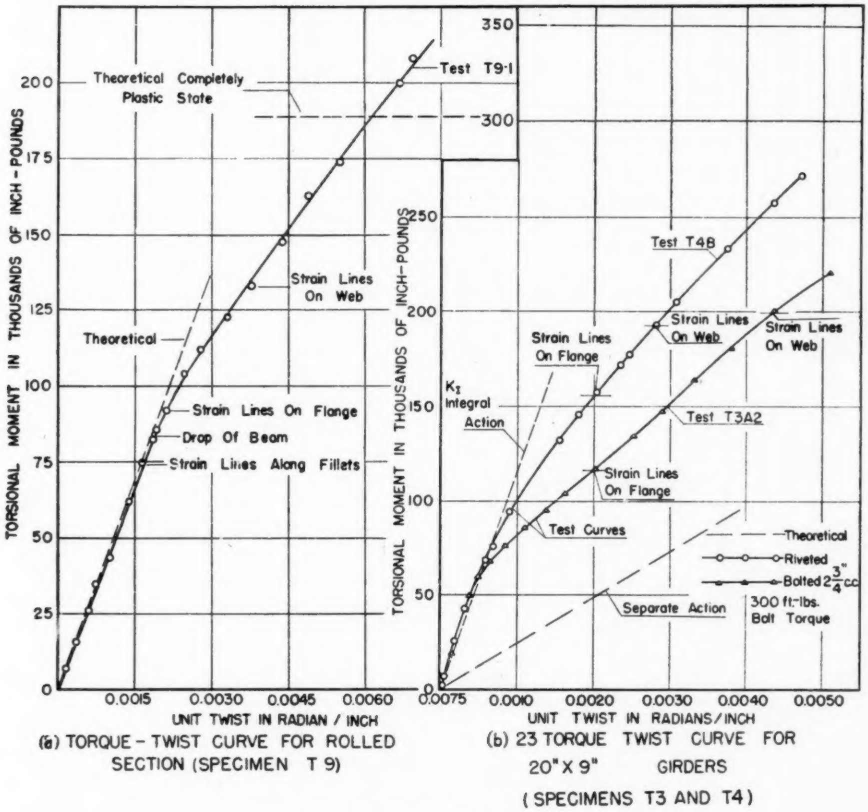


FIG. 15.—TORQUE-TWIST CURVES

tests to study the effect of various factors on the stiffness of bolted girders. In test T3A2, as shown in Fig. 15(*b*), strain lines appeared around the bolt heads at 117,000 in.-lb and on both the flange and web at 210,000 in.-lb. At a torque of about 340,000 in.-lb buckling of the web became noticeable because of the flange shortening effect. The torque-twist curve for specimen T3A2 is compared with that of specimen T4B1 that has the same dimensions except that beam T4B1 is riveted instead of bolted. It may be noticed that the bolted (300 ft-lb bolt torque) and riveted specimens behave more or less the same in the lower load range, but the riveted specimens were relatively stronger in the inelastic range, after initial slip.

In order to accurately determine the torsion constant (K) of this and other test specimens, the elastic range torque-twist curves were drawn to a larger

scale than that of Fig. 15(*b*). From Eq. 2, K is defined as the slope of the torque-twist curve (in the elastic range) divided by G.

The effect of stiffeners on bolted girders was also studied in the test of specimen T6. The test results are plotted in Fig. 16. The shearing stress

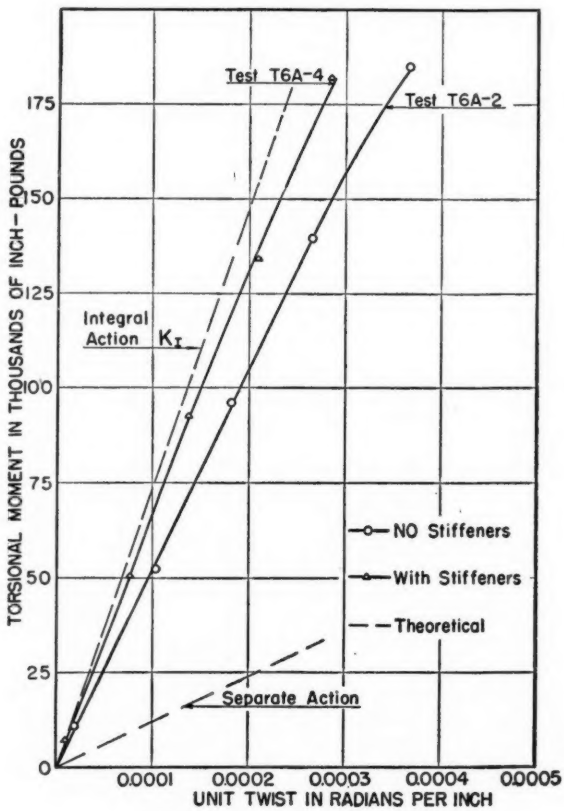


FIG. 16.—EFFECT OF STIFFENERS

distribution in the bolted specimens was similar to that shown by the riveted girders.

Riveted Plate Girders.—Riveted plate girders 20 in. deep were tested to failure. The torque-twist curve for test T1B (with no cover plates) is shown in Fig. 17(*a*). The torque-twist curve for beam T2B (one cover plate) is shown in Fig. 17(*b*). Strain lines first appeared on the flange near the rivet heads at a load of 111,000 in.-lb. At 202,500 in.-lb strain lines started to appear on the web.

The torque-twist curve beam of T4B (with two cover plates) is plotted in Fig. 15(*b*) in comparison with that of a similar bolted girder. The sequence of appearance of strain lines on specimen T4B was similar to that of specimen T2B.

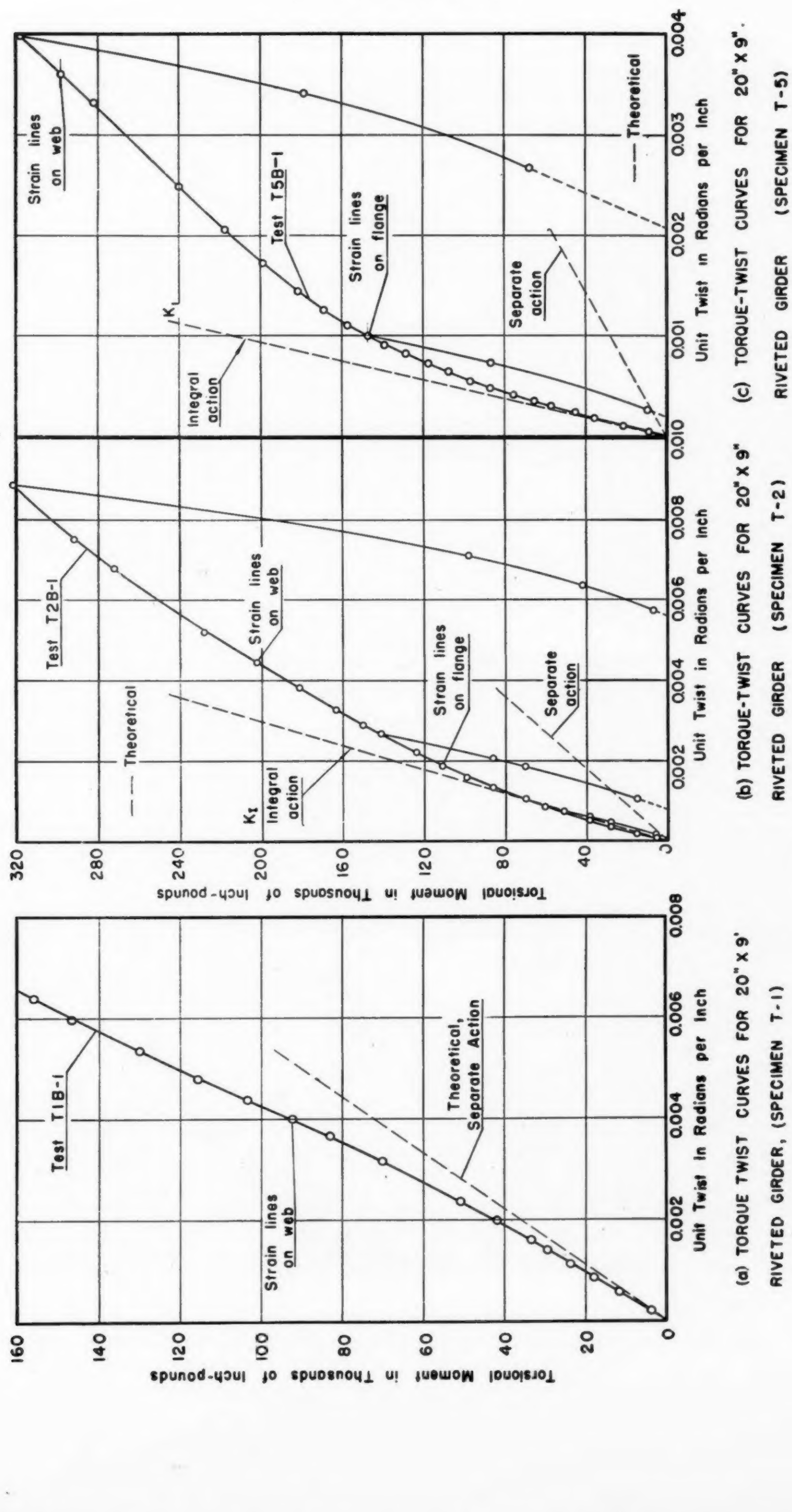


Fig. 17.—Torque-Twist Curves for Riveted Girders

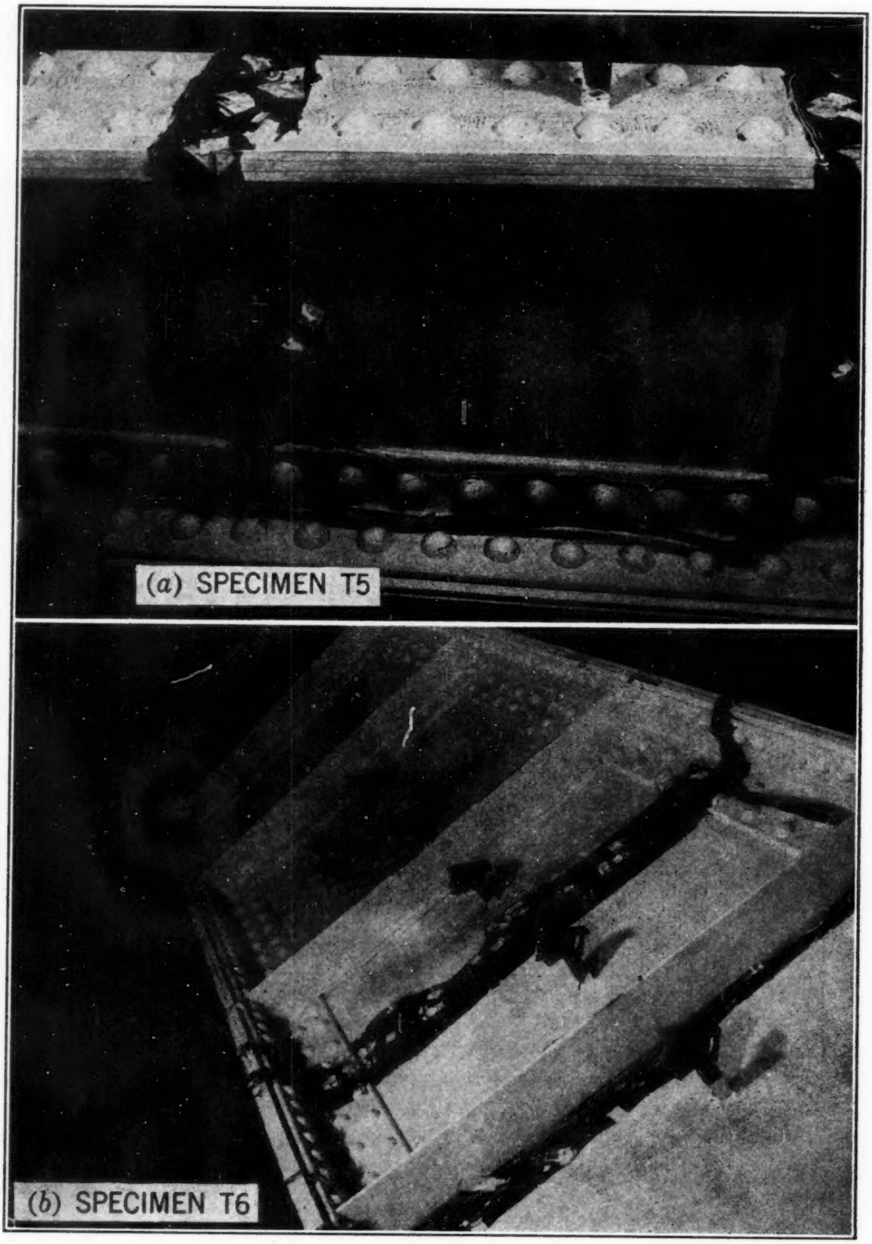


FIG. 18.—STRAIN LINE PATTERNS

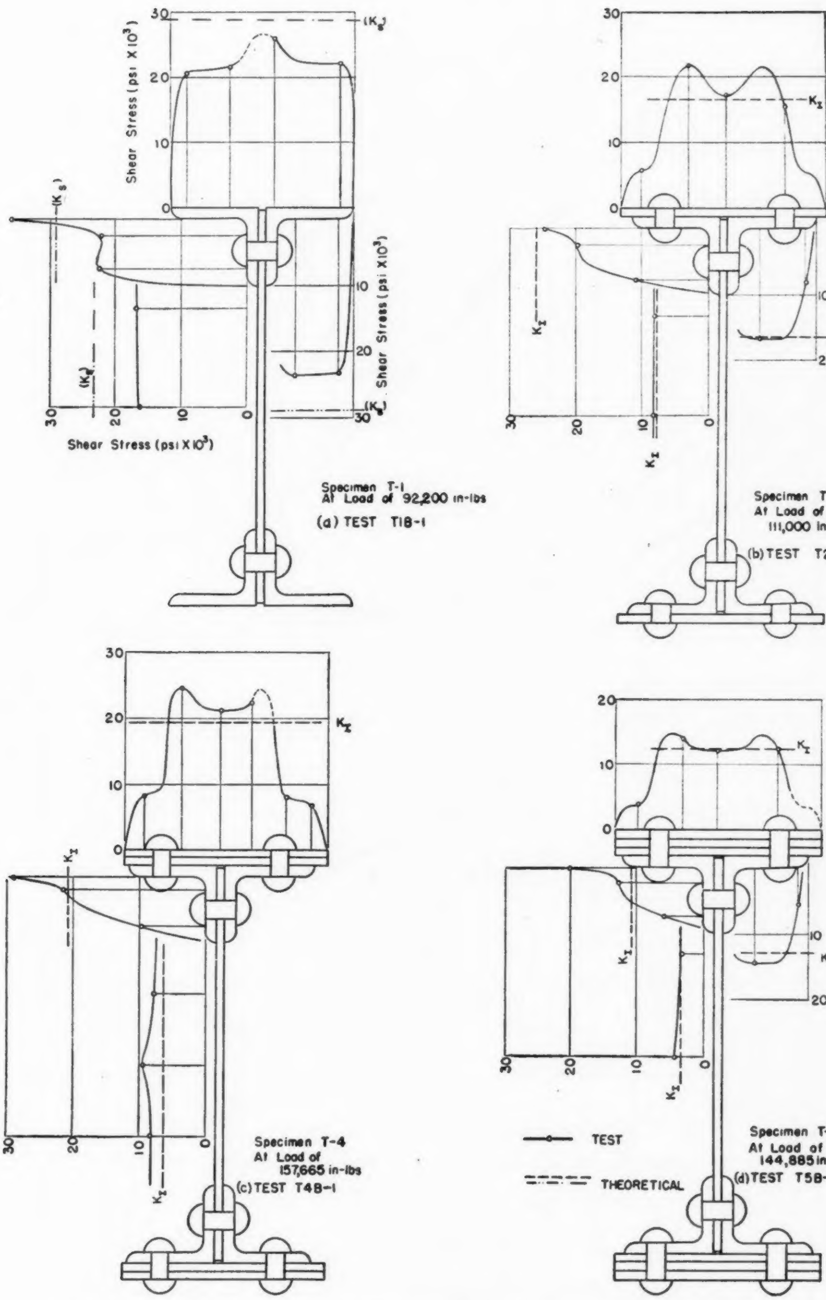


FIG. 19.—RIVETED GIRDER DISTRIBUTION OF SHEAR STRESS IN KIPS PER SQUARE INCH

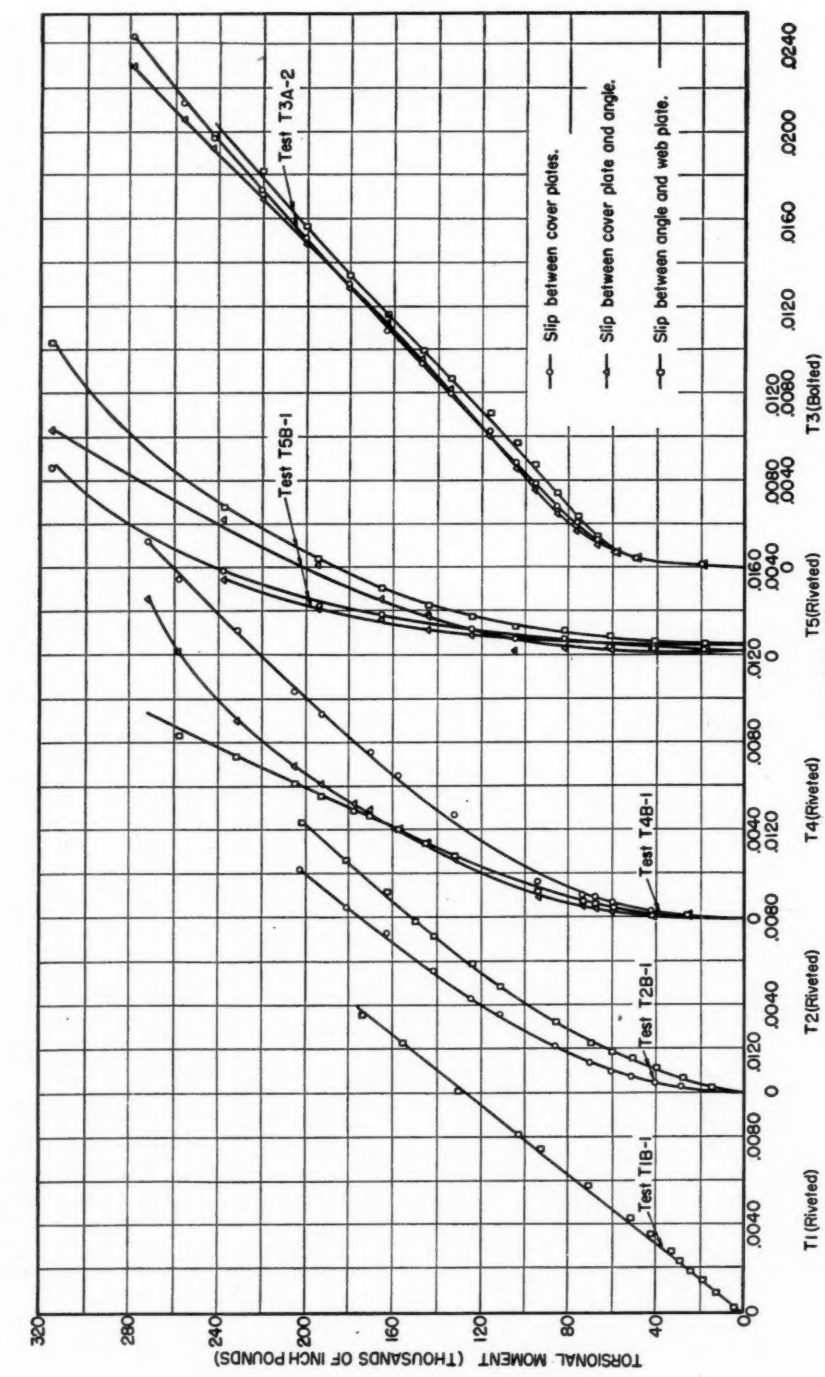


FIG. 20.—VARIATION OF LONGITUDINAL SLIP WITH APPLIED TORQUE FOR RIVETED AND BOLTED GIRDERS

As with all other bolted and riveted specimens, the first strain lines in test T5B (with 3 cover plates) appeared on the flange near the rivet heads. At a load of 240,200 in.-lb the strain lines started to appear on the rivet heads themselves. Strain lines as shown in Fig. 18 appeared on the web when the load reached 298,000 in.-lb. The torque-twist curve is shown in Fig. 17(*c*).

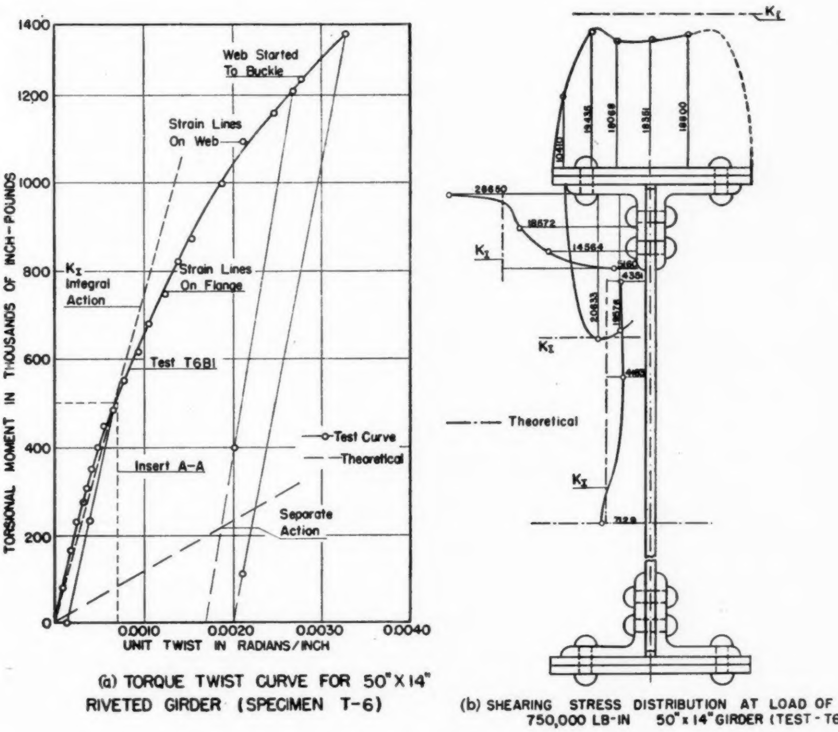


FIG. 21.—CHARACTERISTIC CURVES FOR RIVETED GIRDER

Shear stress distribution at certain loads is shown in Fig. 19 for all the 20-in. riveted girders (T1B, T2B, T4B, and T5B), and longitudinal slip for these specimens is shown in Fig. 20.

The 50-in. deep riveted plate girder with stiffeners (specimen T6B, Fig. 8(*a*)) was tested to failure. The torque-twist curve is shown in Fig. 21(*a*). The first strain line appeared at the flange near the rivet heads at 750,000 in.-lb and then progressed along the flange between the two rows of rivets. This behavior agrees with the assumption made in evaluating the integral action torsion constant, that the portion between rivet lines acts as a solid section. Fig. 18(*b*) shows the strain line pattern after twist when a maximum load of 1,340,000 in.-lb had been applied to the specimen. The strain lines on the web indicate the buckling that occurred. Fig. 21(*b*) shows the shear stress distribution at 750,000 in.-lb torque.

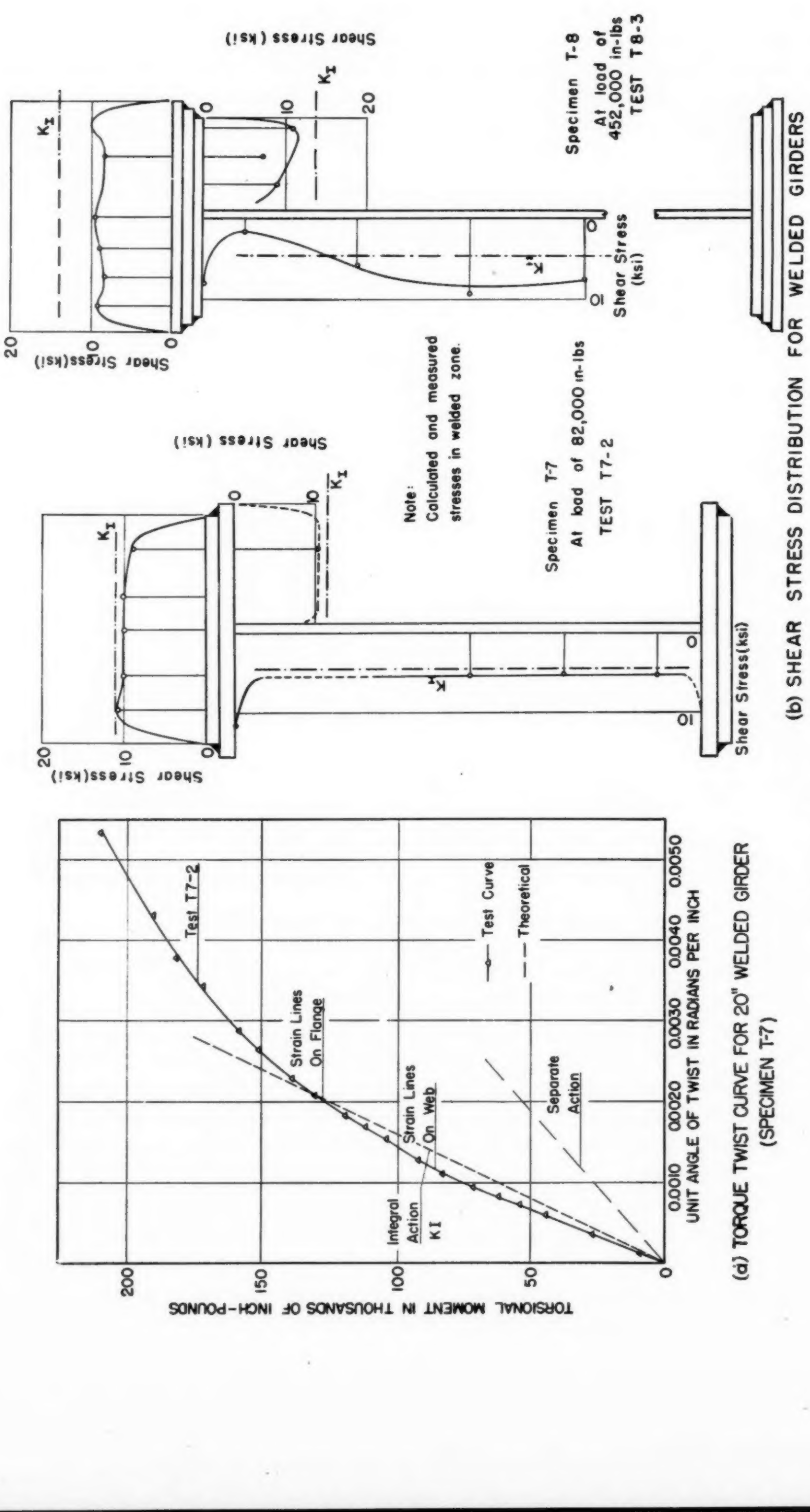


Fig. 22.—Characteristic Curves for a Welded Girder

Welded Girders.—The 20-in. welded girder (specimen T7-2, Fig. 14(*a*)) was first tested in the elastic range with no cover plate, then tested to destruction with one cover plate on each flange. Fig. 22(*a*) is the torque-twist curve for the latter test. The shear stress distribution at 82,000 in.-lb is shown in Fig. 22(*b*).

A 50-in. welded plate girder with no cover plate (specimen T8-3, Fig. 8(*b*)) was first tested in the elastic range with and without stiffeners. The girder with

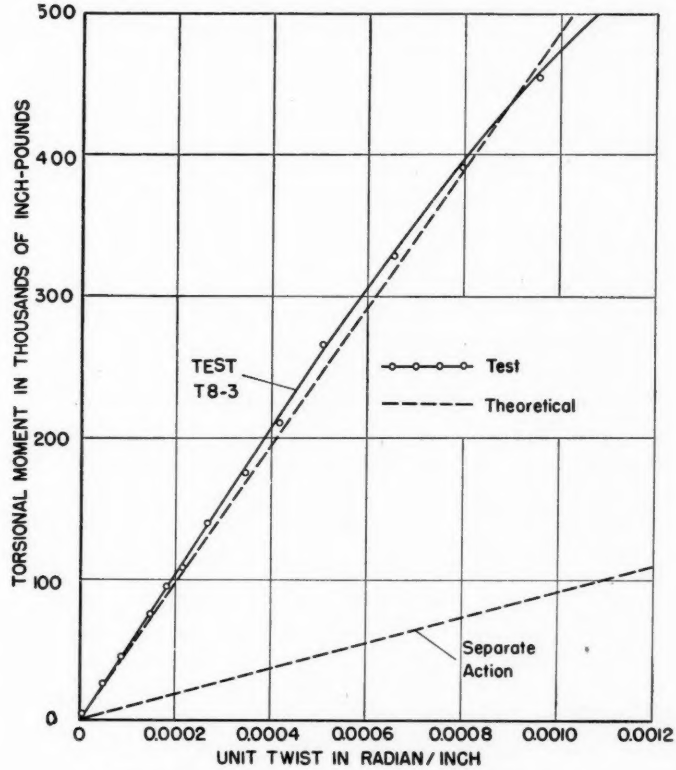


Fig. 23.—Torque-Twist Curve for Welded Girder

web stiffeners and also two cover plates added to each flange was then tested to failure. The torque-twist curve is shown in Fig. 23. Good agreement is noted between the theoretical value of K (K_{eff} as determined by Eq. 22) and the test results. The maximum torque applied was 940,000 in.-lb. At 519,000 in.-lb strain lines first appeared along the inward side of the flange and at 639,000 in.-lb torque, strain lines appeared on both the flange and the web. The buckling of the web became very apparent at a torque of 710,000 in.-lb and at 930,000 in.-lb torque, the welds connecting the cover plates started to break, with a corresponding rapid increase in angle of twist.

The shear stress distribution at 452,000 in.-lb is shown in Fig. 22(*b*) for a location within the zone of the intermittent welds. The flange stresses in this

section are lower than the theoretical although the web stresses are higher, indicating the possibility that in this region of the girder the shape of the section was not being maintained and the flanges were twisting less than the web. Several shear stress measurements were made between the intermittent welds,

TABLE 4.—SUMMARY OF TEST PROGRAM AND COMPARISON WITH THEORETICAL VALUES

Test number	Torsion Constant[a]			Ratio K/K_I	Shear stress τ_y[b] (kips per sq in.)	Initial Yield Moment (In.-Kips)			Moment[d] at a proportional limit (in.-kips)	Ratio		
	Theoretical		Test			Theoretical (Eq. 5)[c]	By strain lines	By strain gages		$\frac{\text{Col. 8}}{\text{Col. 7}}$	$\frac{\text{Col. 9}}{\text{Col. 7}}$	$\frac{\text{Col. 10}}{\text{Col. 7}}$
	K_S	K_I	K									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(a) Bolted Specimens ($\frac{7}{8}$-In. Diameter Bolts at 300 Ft-Lb Bolt Torque)												
P2-5	3.13	11,28	10.70	0.95	....	....	....	....	....	....	....	....
P2-12	6.27	85.60	83.80	0.98	....	....	....	....	....	....	....	....
T2A-3	1.94	5.75	7.42	1.29	....	....	....	....	....	....	....	....
T3A-2	2.28	10.36	10.65	1.03	20.9	174	117	118	60	0.67	0.68	0.35
T5A-3	2.43	18.65	15.43	0.83	....	....	....	....	....	....	....	....
T6A-4	10.22	64.92	59.60	0.92	....	....	....	....	....	....	....	....
(b) Riveted Specimens ($\frac{7}{8}$-In. Diameter Rivets)												
P2-19	6.27	67.25	61.10	0.91	....	....	....	....	....	....	....	....
T1B-1	1.56	1.56	1.86	1.19	20.7	65	82	70	....	1.26	1.08	....
T2B-1	1.94	5.75	6.61	1.15	21.1	140	111	124	56	0.79	0.89	0.40
T4B-1	2.28	10.36	12.20	1.18	20.8	173	158	157	60	0.91	0.91	0.35
T5B-1	2.43	18.65	17.40	0.93	21.1	242	168	198	100	0.69	0.82	0.41
T6B-1	10.22	64.92	73.10	1.13	21.7	738	750	750	530	1.02	1.02	0.72
(c) Welded Girders												
T7B-1	2.34	5.40	6.35	1.18	19.8	77	138	138	100	1.79	1.79	1.30
T8B-1	8.01	42.47[e]	43.40	1.02	20.5	266	519	581	480	1.95	2.18	1.80
(d) Rolled Section												
T9-1	....	4.01[f]	3.83	0.96	19.4	86	86	86	98	1.00	1.00	1.14

[a] Computed values based on measured dimensions. [b] $\tau_y = 0.58\,\sigma_y$ in which σ_y is based on actual tensile tests. [c] $M_y = \frac{K\,\tau_y}{T}$. [d] Moment at 0.00006 radians per in. permanent set. [e] K_{eff} by Eq. 22. [f] Includes hump and end-loss effects.

but not exactly at the midway point. These stresses averaged about 1.5 times the stress in the welded zone. The approximate stress midway between welds, by the tentatively proposed Eq. 24, would be 2.4 times the stress in the welded zone.

Discussion of Test Results

Stiffness of Built-Up Plates and Plate Girders.—In Table 4, the torsion constants obtained by test are compared with the values computed by use of the proposed torsion constant formulas. The equations for K_I, neglecting both

the hump and edge effects, give reasonably good agreement with test results in both the bolted and riveted cases. For specimen T1B, K_S (separate action) is the same as K_I (integral action) because this girder is without cover plate and has only one row of rivets connecting the flange and the web.

For the *riveted or welded girders* (Table 4, Col. 4) the value of K determined by test was never more than 9% less than that determined by formula.

Strength of Built-Up Plates and Plate Girders.—For structural members used in common practice, a unit twist of approximately 0.00006 radians per in. might be allowable. The torsional moments that will cause a permanent set of this value are tabulated in Col. 10, Table 4, thereby defining the approximate range of linear behavior and affording an arbitrary basis of strength comparison.

In the riveted and bolted girder tests the torque-twist curves start to bend before the value of $\tau_{\max}$ reaches the yield point in pure shear. The reverse is true for the rolled section and the welded girders. The torque-twist curves for unloading are nearly straight lines, approximately parallel to the initial straight portion of the torque-twist curve.

Col. 6, Table 4, gives the shear yield of the material as calculated from tensile coupon tests for the various specimens.

By using Eq. 5 and the coupon strength of the material, the torsional moment at initial yield can be predicted and these moments for all the plate girders are listed in Table 4, Col. 7. The moments at the initial yield, as indicated by strain lines (Col. 8) and by strain gages (Col. 9) are also tabulated for comparison. Cols. 11 and 12 show the relationship of these moments to those predicted by the tensile coupon tests and Eq. 5. The initial yield torsional moments as determined by strain lines and strain gages are similar in most of the tests.

Col. 10 lists the torsional moment at an arbitrary degree of permanent twist (0.00006 radians per in.) By examination of Col. 13 it is seen that for bolted and riveted girders, this amount of set develops at 35 to 72% of the theoretical moment for initial yield. This set is caused by two factors: (1) The rivet or bolt pitch used was usually greater than that required by Eq. 11 and (2) the working value of rivets, as permitted by AISC, is very close to the rivet loads that cause initial slip. Cols. 11 and 12 show that (omitting test T1B-1) actual yield occurred at moment loads varying from 67 to 102% of that predicted by Eq. 5 with early rivet or bolt slip undoubtedly being a contributing factor to the shear stresses that were higher than predicted. Test T1B-1 showed different behavior from the other riveted or bolted specimens. The slope of the torque-twist curve in this test increased initially with an increase of moment. Apparently, behavior is initially very nearly that of separate action, as assumed, but the interfaces between flange angles and web plate may develop greater friction because of binding as twist develops, thus approaching partial integral behavior.

The welded girders exhibited quite different behavior from the riveted or bolted specimens. The calculated values of moment at initial yield are based on separate action, because of the fact that the welds are intermittent and spaced at a distance of more than 0.4 b apart. This is probably an over-conservative approximation. Better agreement between theory and test would probably

have been obtained had fully continuous welds been used and these should be required if maximum available torsional strength is required. The behavior of these welded girders, underdesigned as they were for torsion, shows their marked superiority in this respect to similar riveted girders.

The rolled section exhibited excellent agreement between theory and test, as might be expected, since in this case integral behavior is inherent in this section and not a characteristic that is approximately brought about by rivets or welds. Similar agreement should be obtained for welded girders with fully continuous welds of adequate strength interconnecting the edges of all cover plates.

For the shear stress distribution on the surface of the flange and web, the integral torsion constant (K_I), used in Eq. 5, neglecting both the hump and edge effects, is in fairly good agreement with test results.

The testing program emphasized the factors that affect the torsional behavior of built-up structural members. The most important of these are: (*a*) The tension in the bolts; (*b*) the method of driving rivets; (*c*) the bolt or rivet gage line location; (*d*) the pitch of the bolts or rivets; and (*e*) the presence of stiffeners.

(*a*) *Tension in Bolts.*—From Table 2, it is seen that the bolt tension directly affects the bolt values in friction, that in turn are required to supply the longitudinal shearing stresses over the length p in the interface. Slip between different components will occur if the resultant of the latter stresses is greater than the bolt value in friction. Therefore, in a given design, the tension in the bolts will determine at what torsional moment appreciable slip will occur and the corresponding point of departure from straight-line relationship in the torque-twist curve.

(*b*) *Method of Driving Rivets.*—The rivet value in friction varies with the method of driving. If this value is too low, slip may occur very early. Test results indicate that the $\frac{7}{8}$-in. rivets, driven by ordinary shop practice, correspond to $\frac{7}{8}$-in. high strength bolts, without washers, tightened to 300 ft-lb bolt torque.

(*c*) *Gage Line Locations of Bolts and Rivets.*—In evaluating the integral action torsion constant, the assemblage is assumed to act as an equivalent solid section between the outer rows of rivets or bolts. This assumption agrees very well with the test results. Therefore, the gage line location of bolts and rivets affects both the strength and stiffness of built-up members. If possible, the rivet (or bolt) lines should be located near the edge of the built-up member in order to obtain a torsionally stronger and stiffer member.

(*d*) *Pitch of Rivets or Bolts.*—The effect of rivet and bolt pitch on the stiffness and strength of built-up structural members has been approximately evaluated. If the pitch is larger than $p' = A + T$ (the pitch required for longitudinal continuity), the torsional constant (K) will be reduced according to Eqs. 17 and 22 and the maximum shear stress will be increased in comparison with full integral behavior. Likewise, the torsion constant is reduced and the shear stress raised if intermittent welds are used in welded girders.

(*e*) *Stiffeners.*—Stiffeners have little effect on the strength and stiffness of built-up members under uniform torsion in the elastic range. The riveted girders with stiffeners are somewhat stiffer than those without stiffeners. The

effect of stiffeners on welded girders is not appreciable. An important function of stiffeners is to tie the flange and web together in deep girders to assure that the flange and the web will twist through the same angle. In other words, the stiffeners serve to maintain the shape of the cross section, an important function at points of torsional load application.

Summary

Equations for evaluating the torsion constant of built-up plates and plate girders are developed and illustrative examples presented. Eqs. 14, 26, and 28, for built-up plates, riveted and bolted plate girders, and welded plate girders, respectively, are recommended for practical design purposes.

The pitch of rivets or bolts affects both the stiffness and strength of built-up structural members in torsion. For strength, the pitch should be designed by using Eqs. 11 and 12 and for longitudinal continuity the pitch should not be greater than $p' = A + T$. If a pitch greater than p' is used, the torsion constant should be calculated by Eqs. 17 and 22 and shear stress computed by Eq. 24. The reduced strength and stiffness of welded girders with intermittent welds may be evaluated in a manner similar to that for riveted girders having a pitch greater than p', the effective clamping distance, which in this case corresponds to the length of the individual weld.

The typical shear stress distributions in bolted and riveted plate girders under torsion are shown in Figs. 19, 21, and 22. The maximum shear stress in the girder sections tested occurs in the fillet between flange and web. The next highest stress occurs in the flange near the rivet (or bolt) head. The stress concentrations in the fillets have little effect on the over-all torsional behavior. Eq. 5 is in good agreement with measured shear stresses in the flange and web away from the fillets.

The slip between different components of the built-up members under torsion is discussed. Eq. 6 determines the amount of slip between the corners of loosely bolted built-up plates. The tightness of bolts or the method of driving rivets determines at what torsional moment slip will occur and at what point the torque-twist curve will depart from the straight line relationship.

Stiffeners in deep riveted plate girders increase the torsional stiffness somewhat and may be required in deep girders to maintain the shape of the cross section.

Acknowledgment

This project was carried out in the Fritz Engineering Laboratory of the Lehigh University's Department of Civil Engineering and Mechanics, which is under the chairmanship of W. J. Eney. Facilities for the program were made available as well as the assistance of Paul Kaar, Kenneth Harpel, and other staff members.

The building of the special torsion testing machine required for this program received additional financing through the support of the Institute of Research of Lehigh University together with aid from the Pennsylvania State Department of Highways in cooperation with the United States Bureau of Public Roads. The program itself received major additional financing by the Research Corpor-

ation of New York City, N. Y., and the Lehigh Structural Steel Company, the latter firm having donated most of the test specimens. The Sub-committee on Torsion of the ASCE Applied Mechanics Committee aided in planning and guiding the test program.

This paper is based on a complete report of the test program that was prepared by Mr. Chang as a thesis in partial fulfilment of requirements for a doctorate degree at Lehigh University in 1950. The title of this thesis is "Torsion of Built-up Structural Members" and it is available on loan from the university.